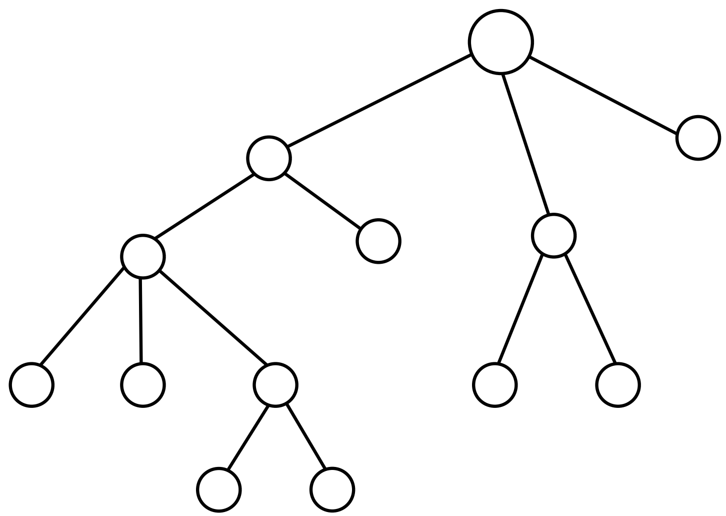
Sampling Based Algorithms for Quantile Computation in Sensor Networks

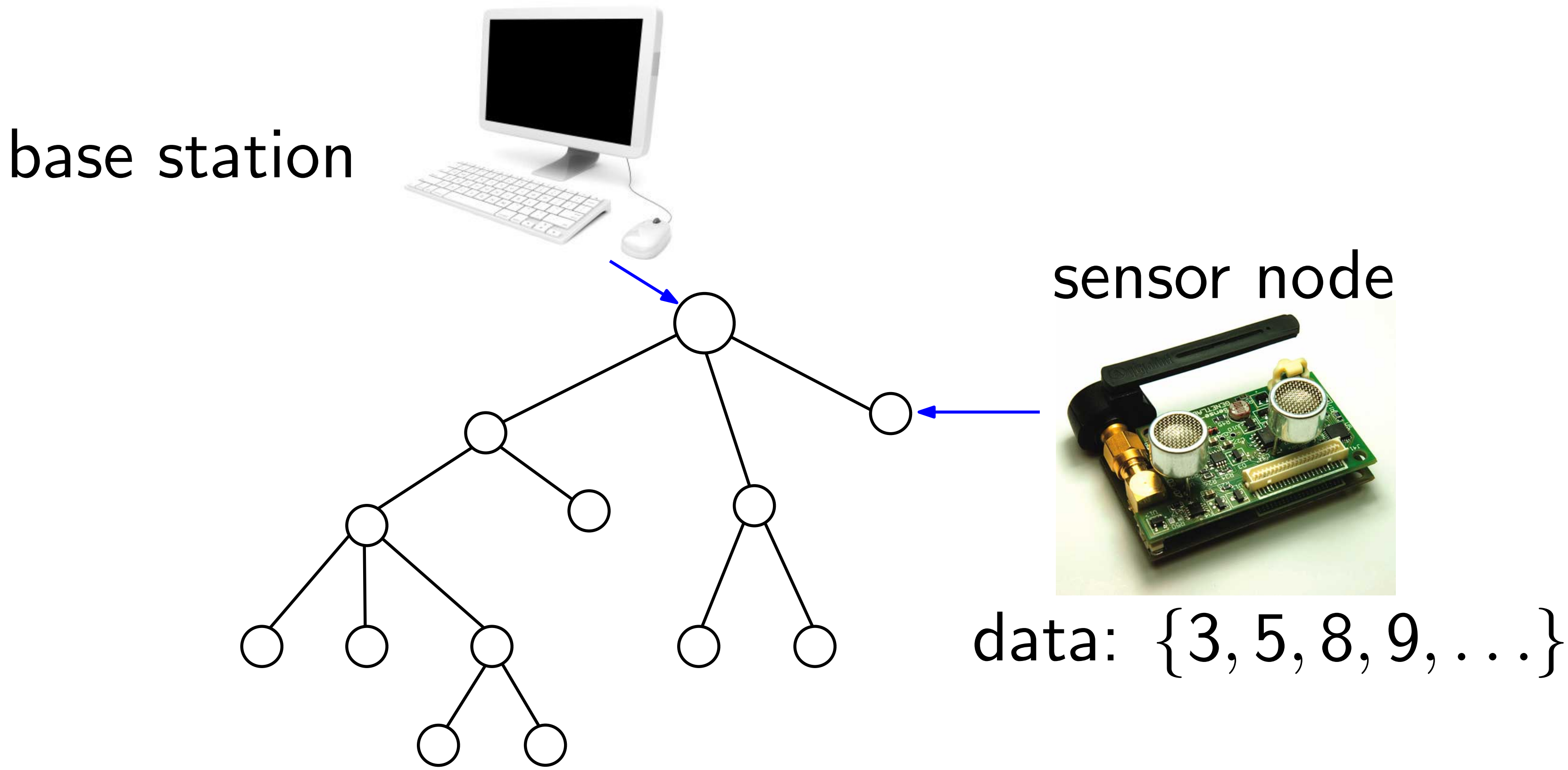


Zengfeng Huang, Lu Wang, Ke Yi, Yunhao Liu

Hong Kong University of Science and Technology

SIGMOD'11

Wireless Sensor Networks



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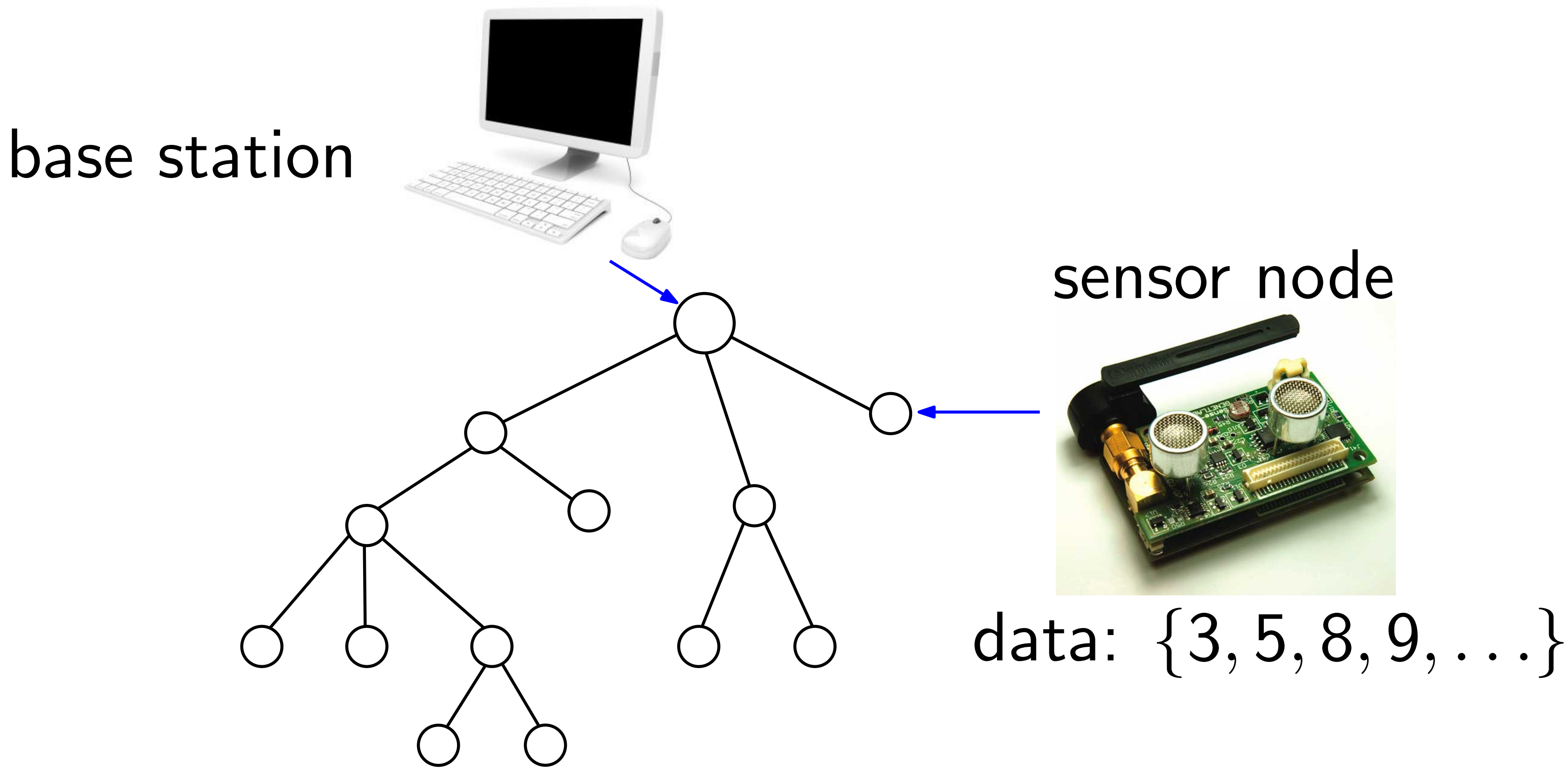


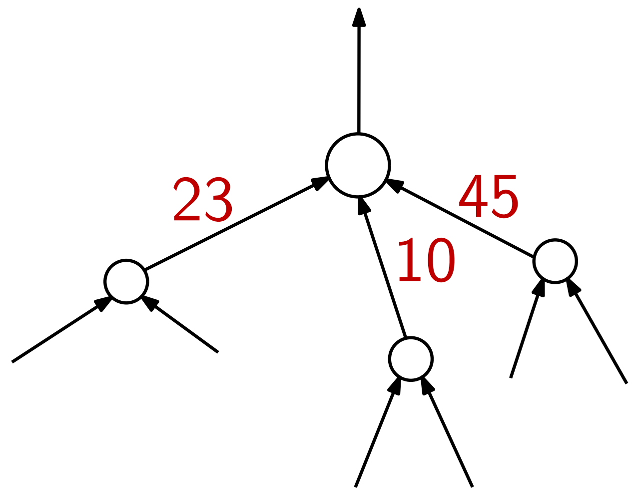
Assume for this talk:

- The network is a tree (may not be balanced).
- The tree has already been built.

Data Aggregation

Goal: Get all the data in an "aggregated form" to save communication.

Example: sum



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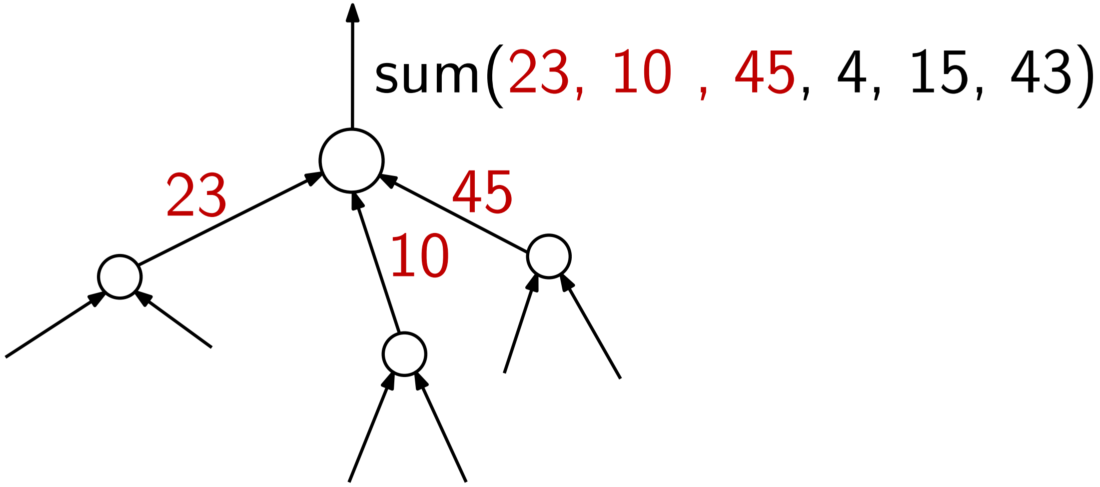

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Also works for: max
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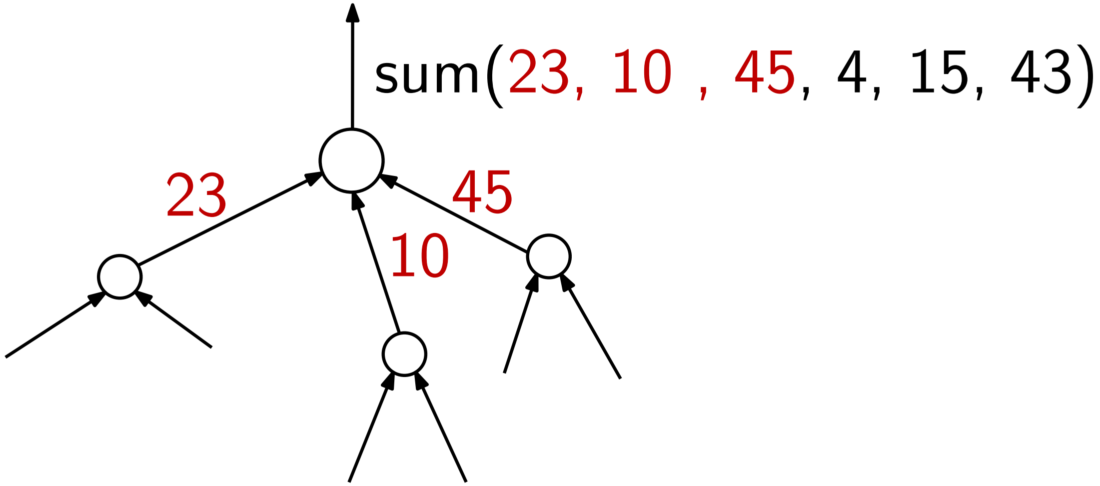
average



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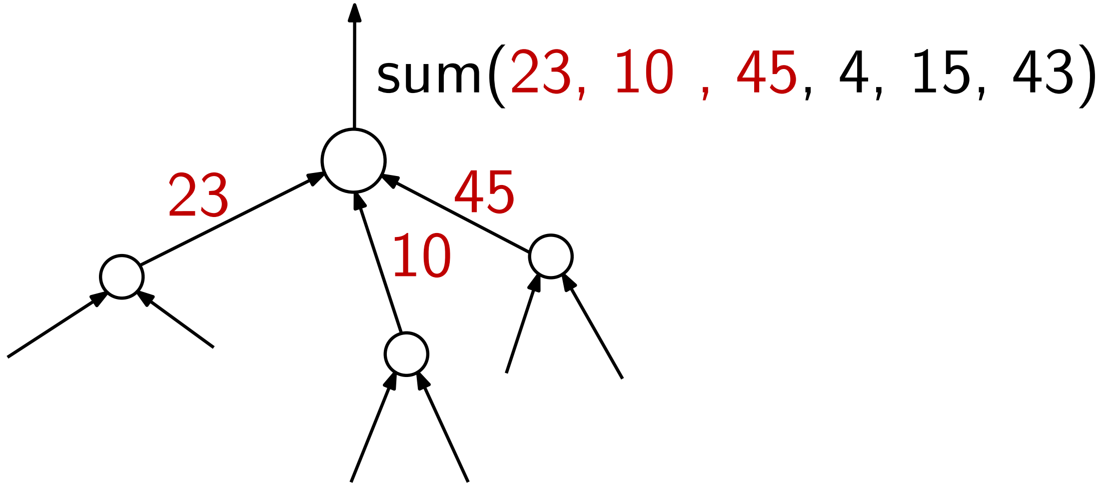


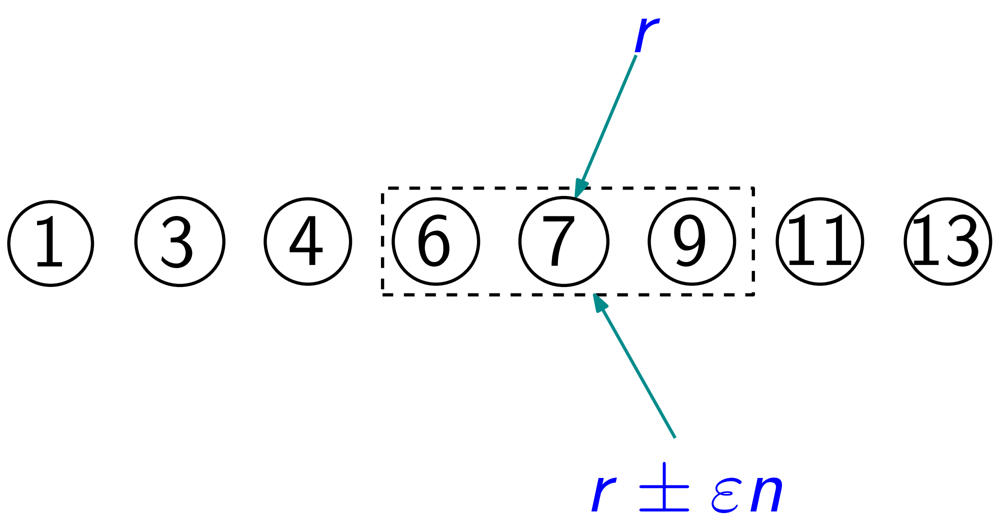
The Decomposable Property

A function f is decomposable if there exists some "combine" function g, such that for any two multisets A, B,

$$f(A \uplus B) = g(f(A), f(B))$$

Quantiles

In a set of n values, the (r/n)-quantile is the value ranked at r. The 0.5-quantile is the median.



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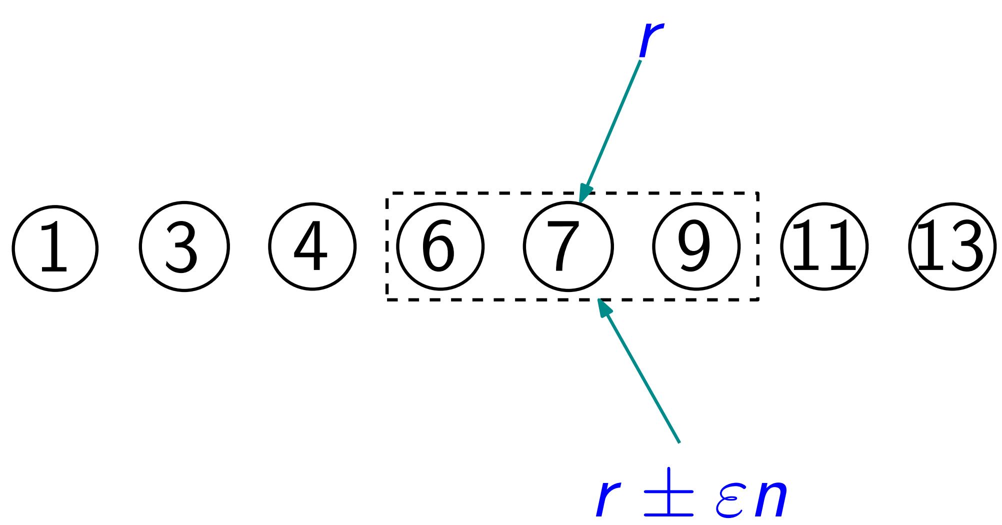

An ε-approximate (r/n)-quantile is any value ranked between $[r - \varepsilon n, r + \varepsilon n]$.

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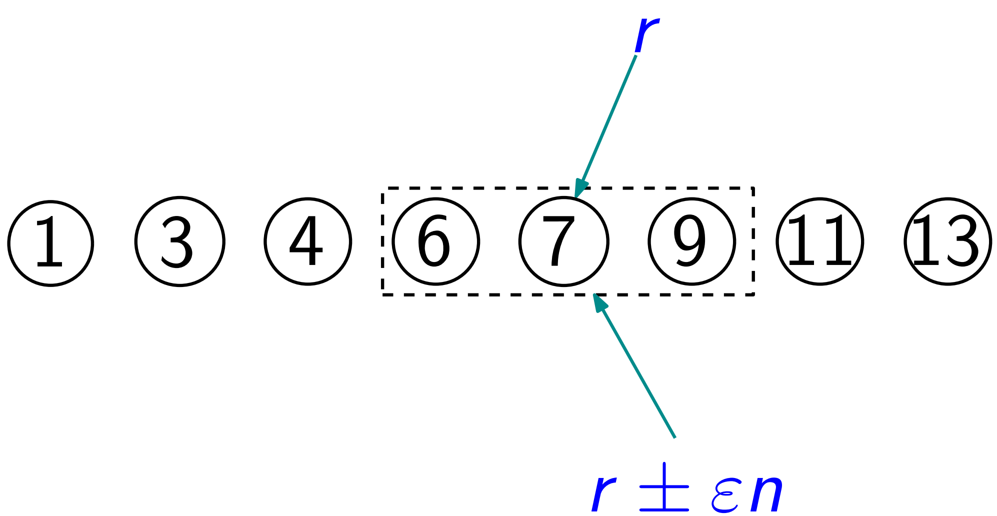

An ε-approximate (r/n)-quantile is any value ranked between $[r - \varepsilon n, r + \varepsilon n]$.

Problem: Quantiles are NOT decomposable

Quantile Summaries

Previous solution: Design a quantile summary that is decomposable, and any (ε-approximate) quantile can be extracted from this summary

	summary size	
q-digest [Shrivastava et al. '04] 272 citations	$O(\frac{1}{\varepsilon}\log u)$	
GK [Greenwald, Khanna '04] 102 citations	$O(\frac{1}{\varepsilon}\log^2 n)$	

n: total data size.

ε: error. $10^{-2} - 10^{-4}$

k: number of nodes. $100 \sim 10000$

h: height of the tree. $\log k \sim \sqrt{k}$

u: size of universe. $\log u = 32$

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New	$O(\frac{1}{\varepsilon}\log k)$	$O(\frac{\sqrt{kh}}{\varepsilon})$

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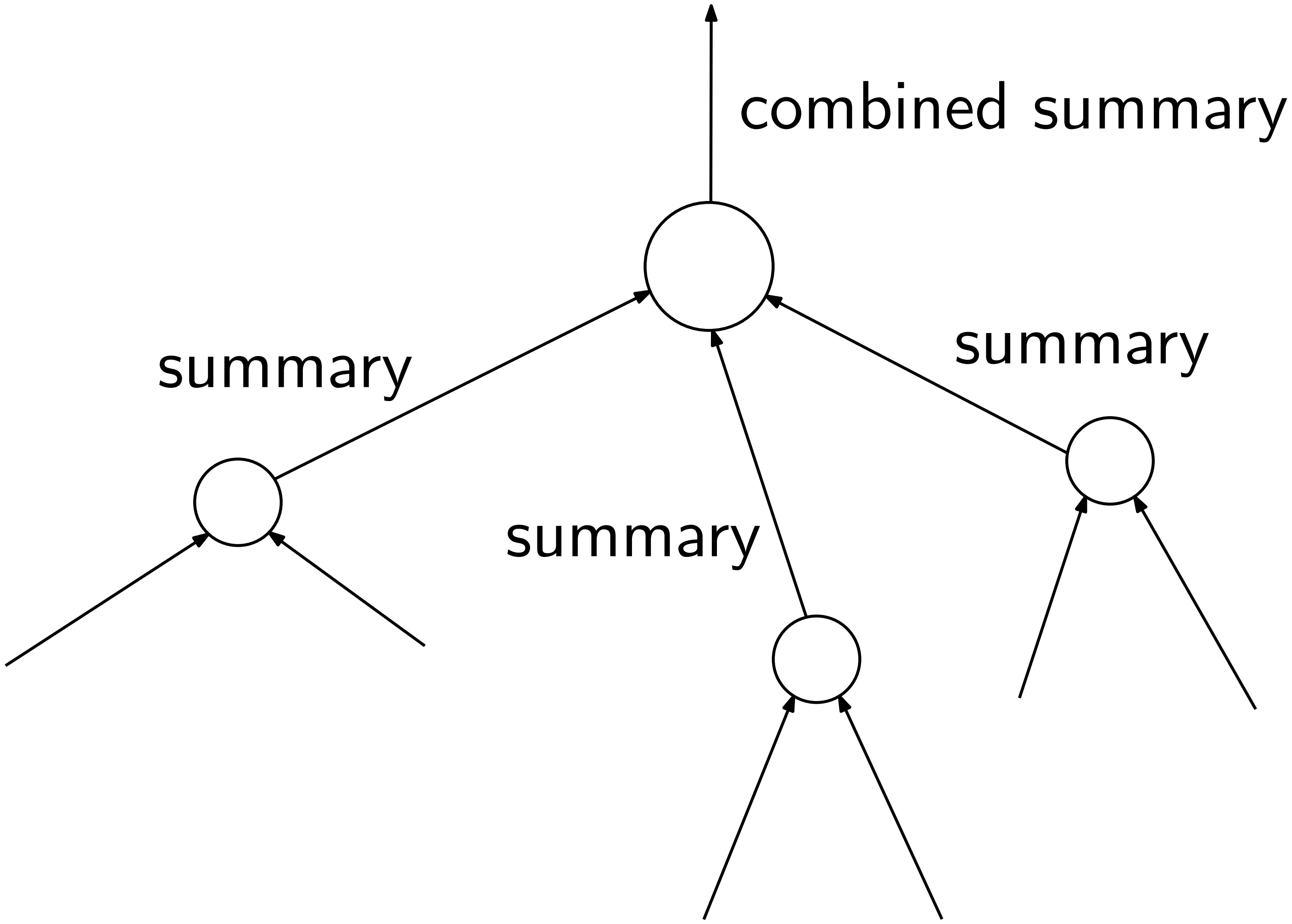
Our Approach



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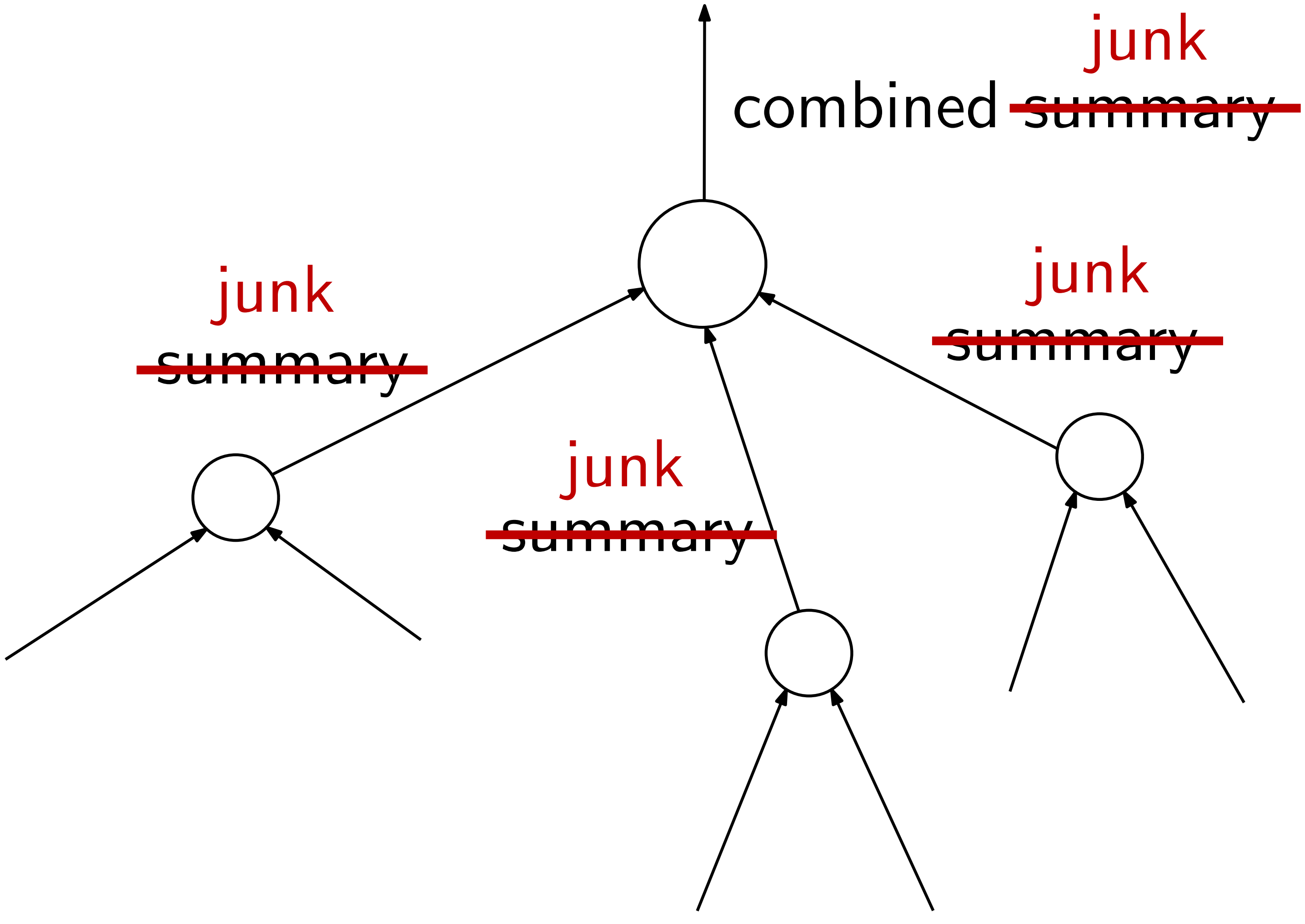

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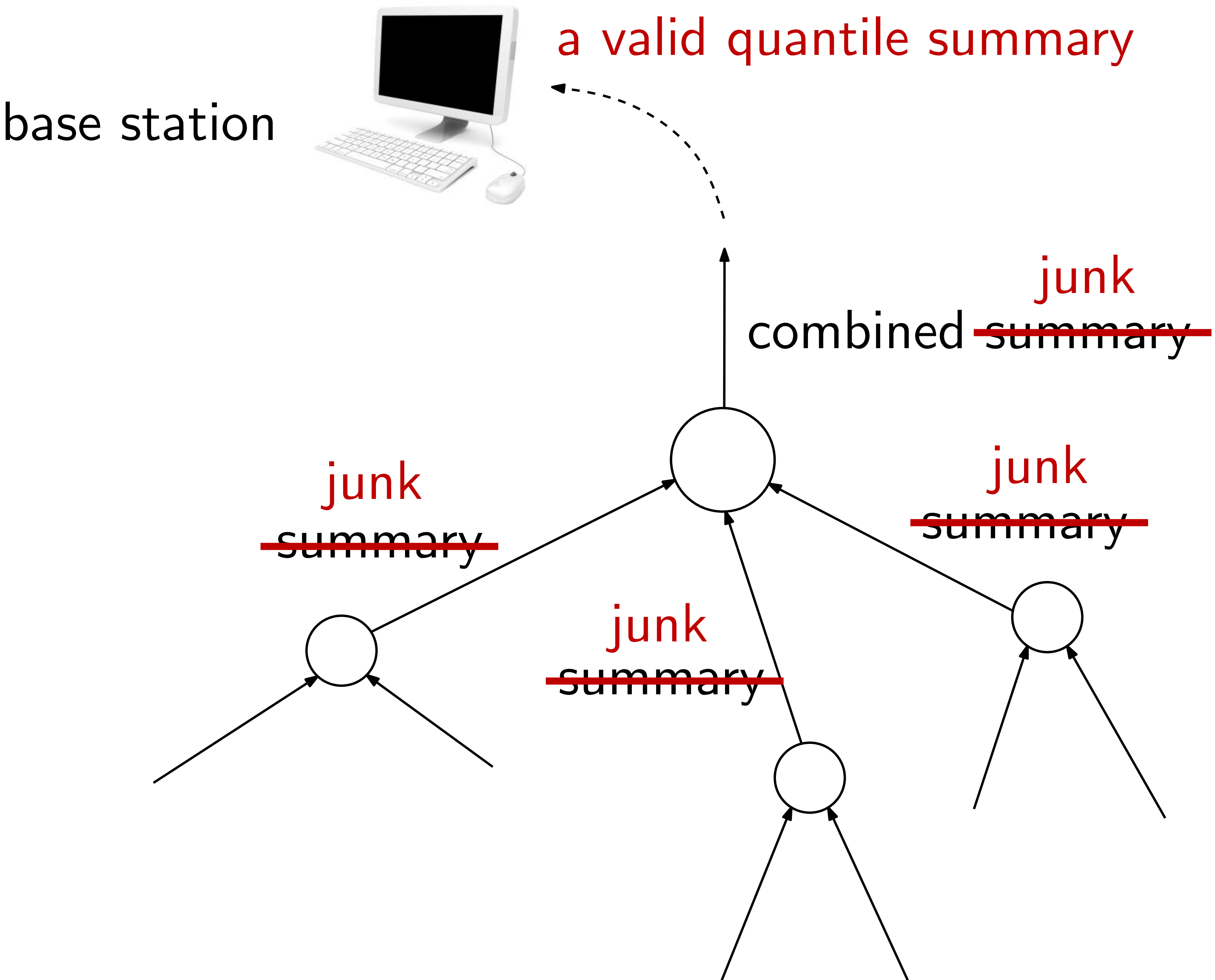
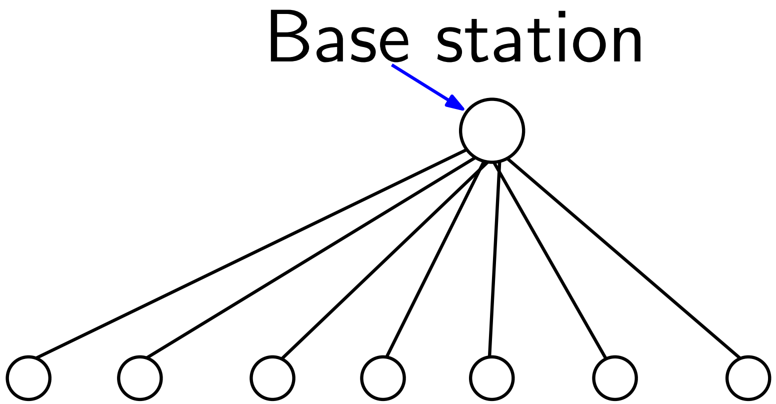
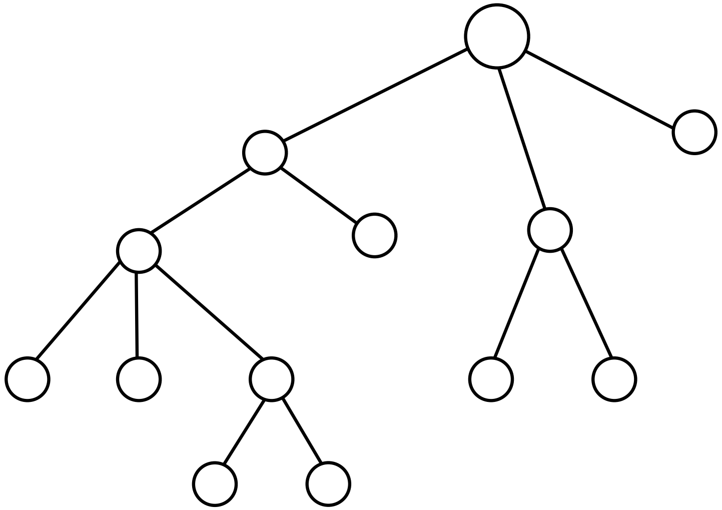


Roadmap

- Flat model



- Tree model



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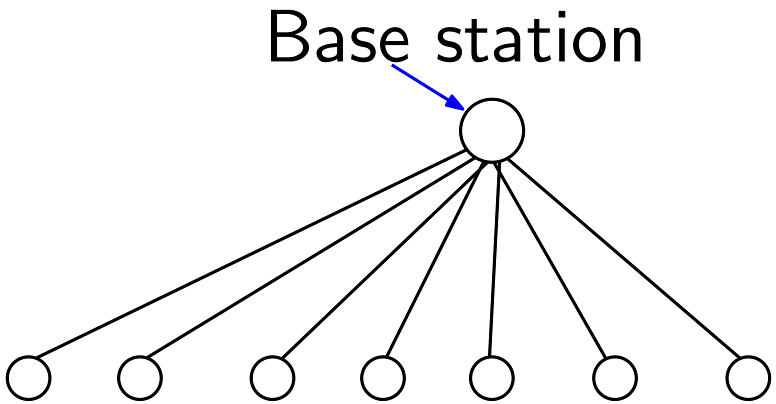

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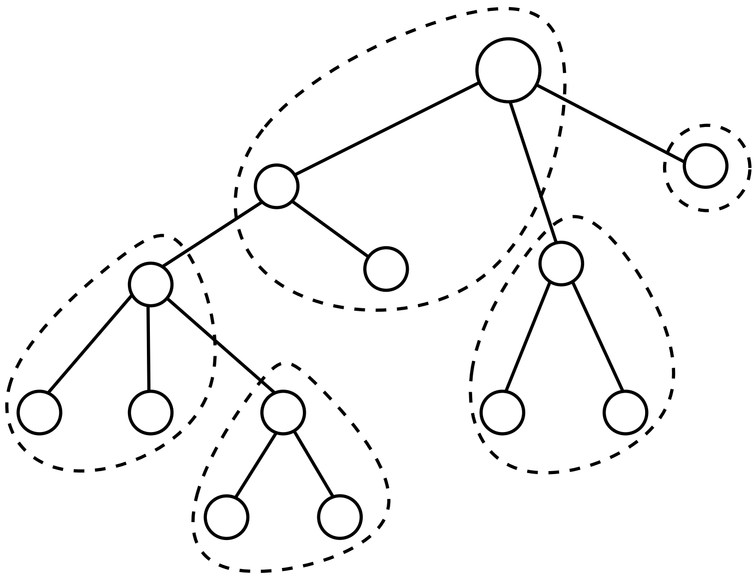

- Partitioned tree

Roadmap

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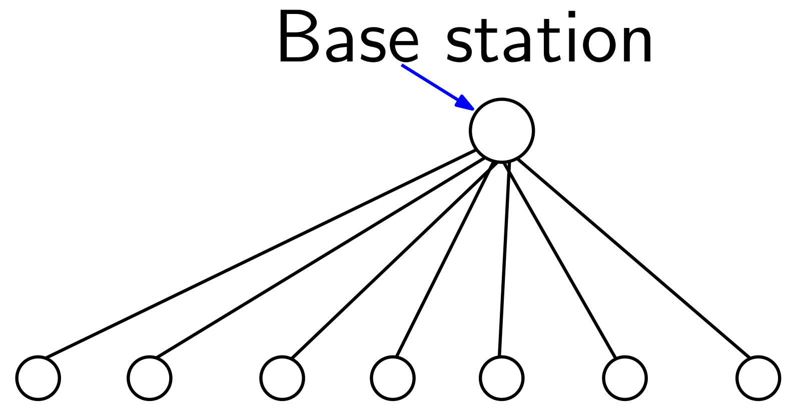

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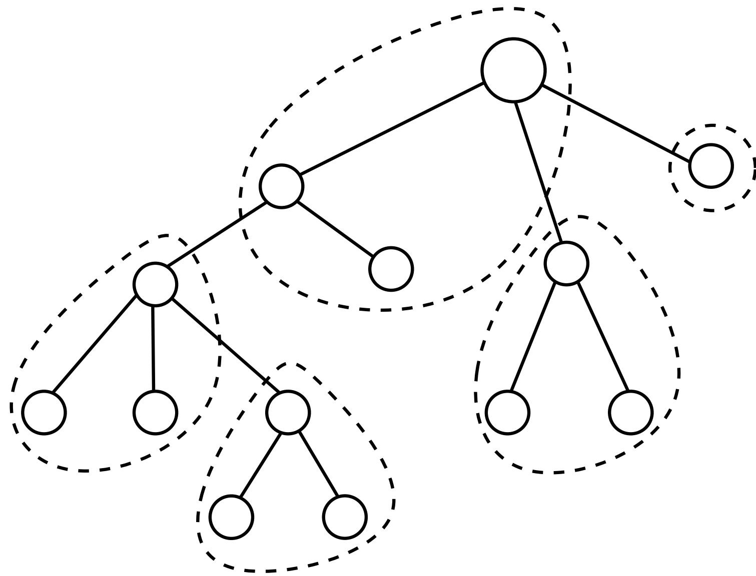


- Partitioned tree

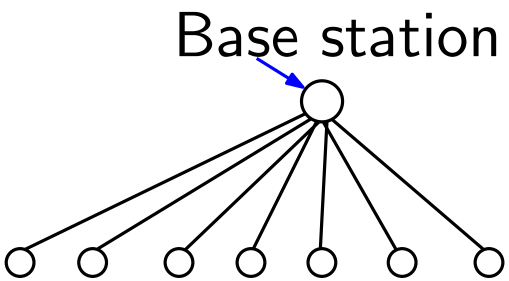
value-to-rank queries

10 $\quad r(10) = 7$

1 3 4 6 7 9 11 13

Return any number within $r(10) \pm \varepsilon n$

The Flat Model: Algorithm



The algorithm for each node

Sample each value with probabiltiy p

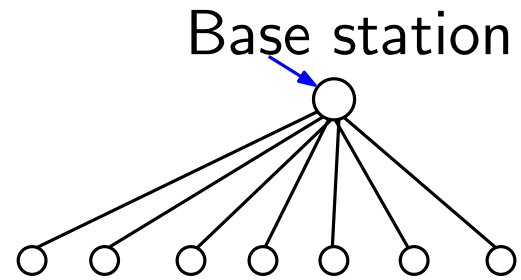
1 3 4 6 7 9 11 13 16 26 21 24

$(3, 2)$ $(7, 5)$ $(13, 8)$ $(26, 10)$

Compute local ranks

Base station

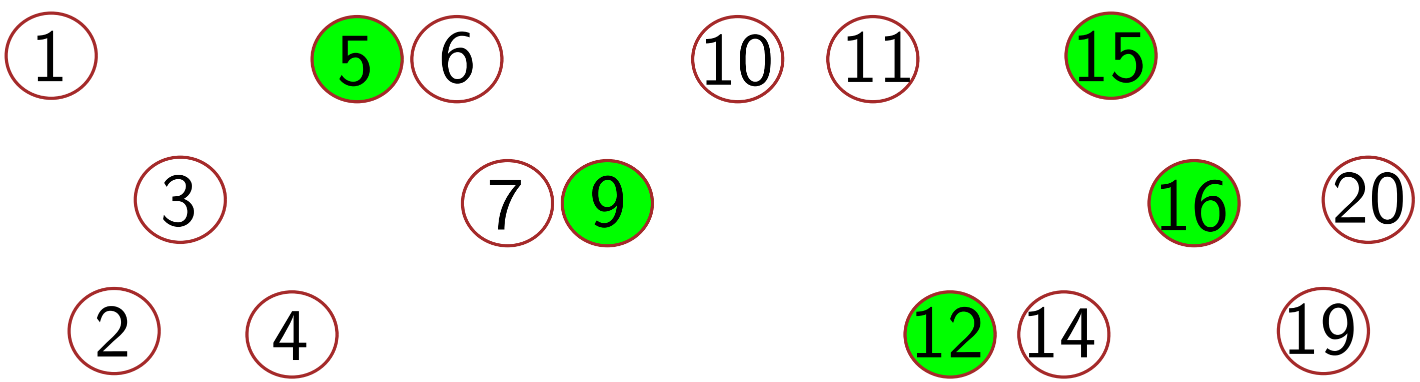
The Flat Model: Algorithm



At the base station:

Answering value-to-rank query

Given any value x, estimates its rank $r(x)$



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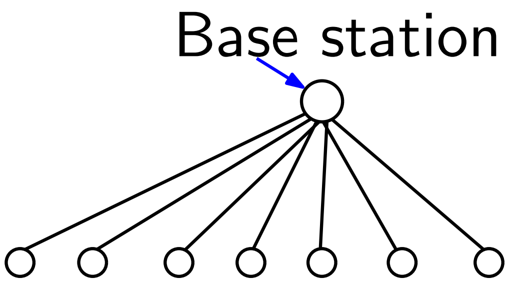

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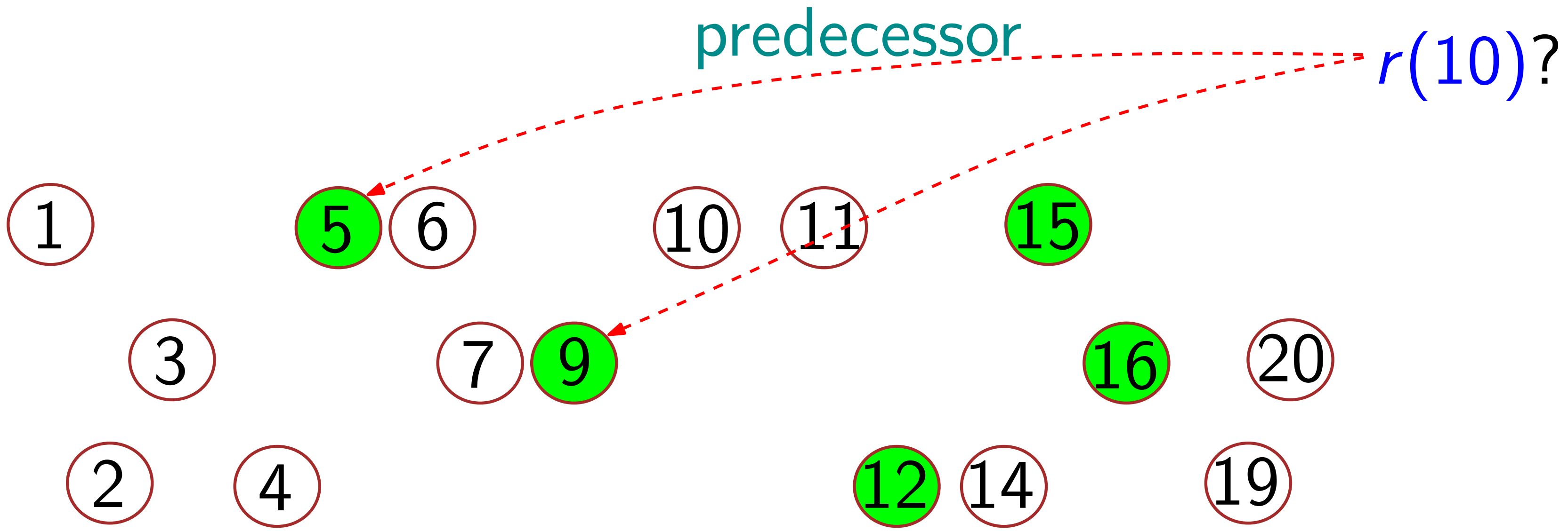

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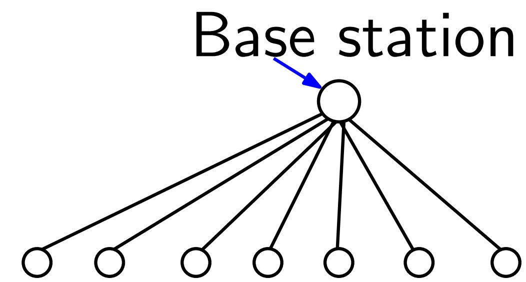

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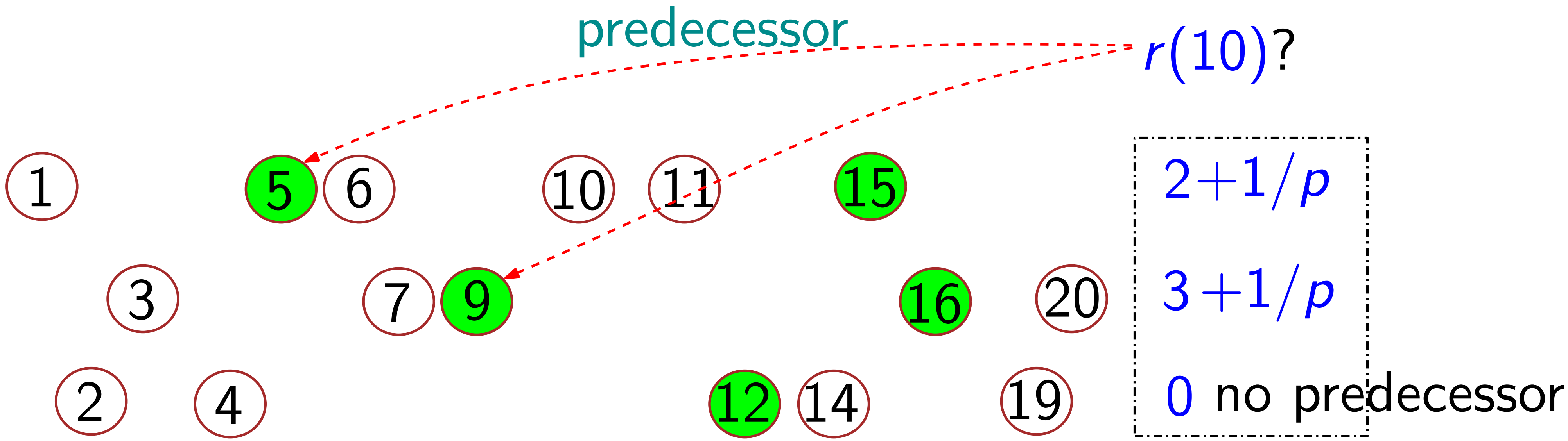

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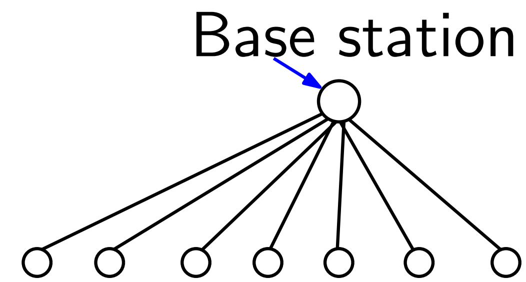

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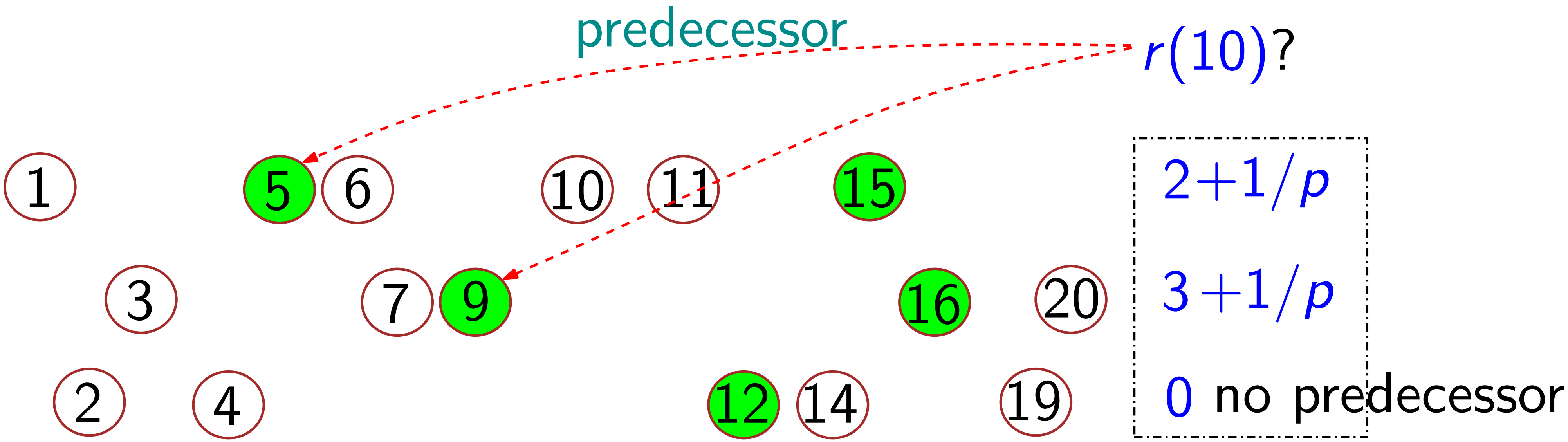

$\hat{r}(10) = 5 + 2/p$

The Flat Model: Correctness

Will show: $\hat{r}(x)$ is an unbiased estimator of $r(x)$ with standard deviation εn.

$r(10)$?

1 5 6 10 11 15

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Follows a geometric distribution (almost)

$\mathrm{E}[?] = 1/p$ $\mathrm{Var}[?] \leq 1/p^2$

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5 ? 15

Follows a geometric distribution (almost)

$\mathrm{E}[?] = 1/p$ $\mathrm{Var}[?] \leq 1/p^2$

Set $p = \frac{\sqrt{k}}{\varepsilon n}$

$\mathrm{Var}[\hat{r}(x)] \leq k/p^2 = (\varepsilon n)^2$

The Flat Model: Communication Cost

- Total cost: $np = \sqrt{k}/\varepsilon$ in expectation
- Max individual node cost: $O(\sqrt{k}/\varepsilon)$

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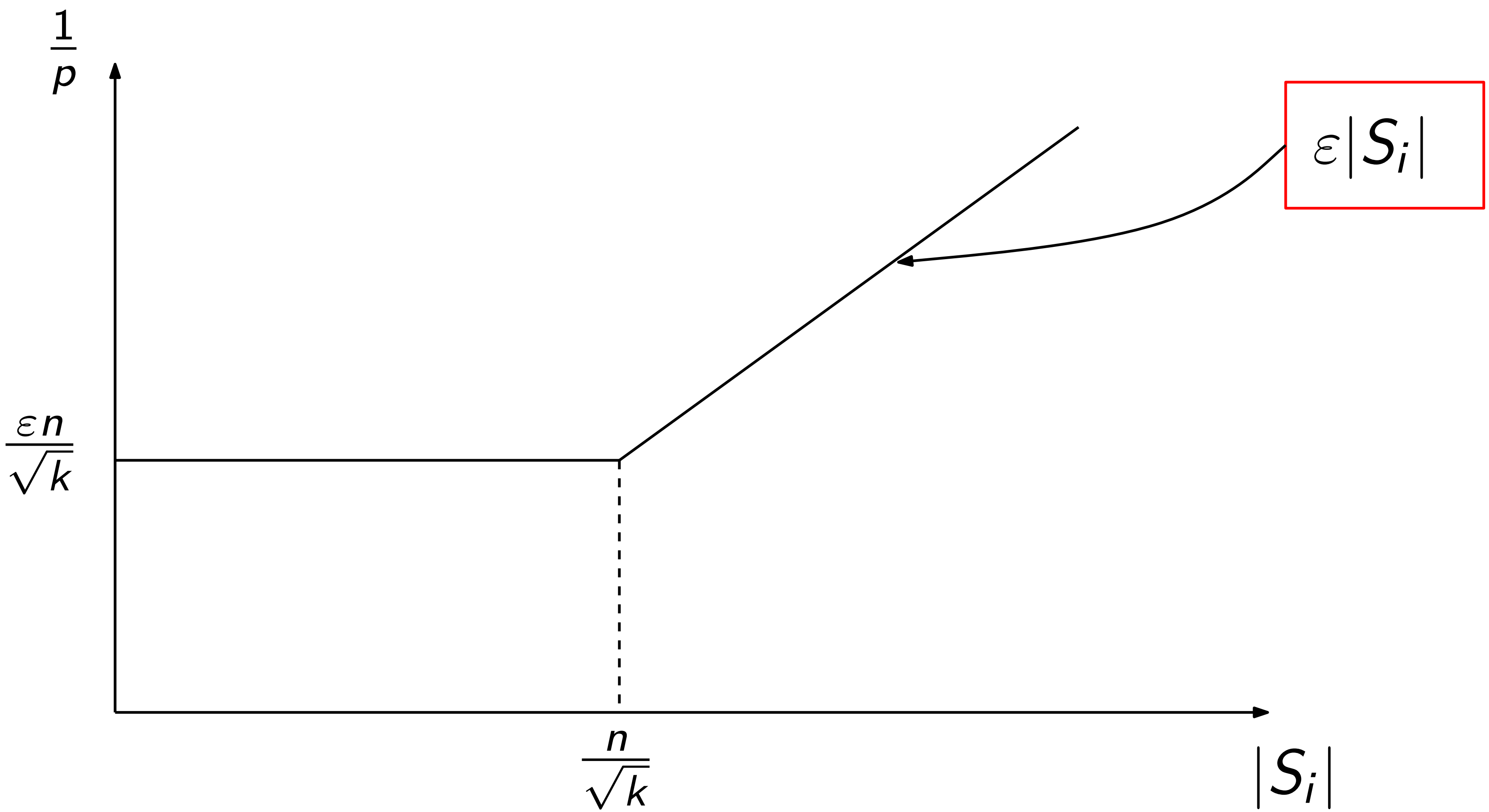
  Reduce to $O(1/\varepsilon)$

The Flat Model: Communication Cost

Let S_i be the set of data collected at node i



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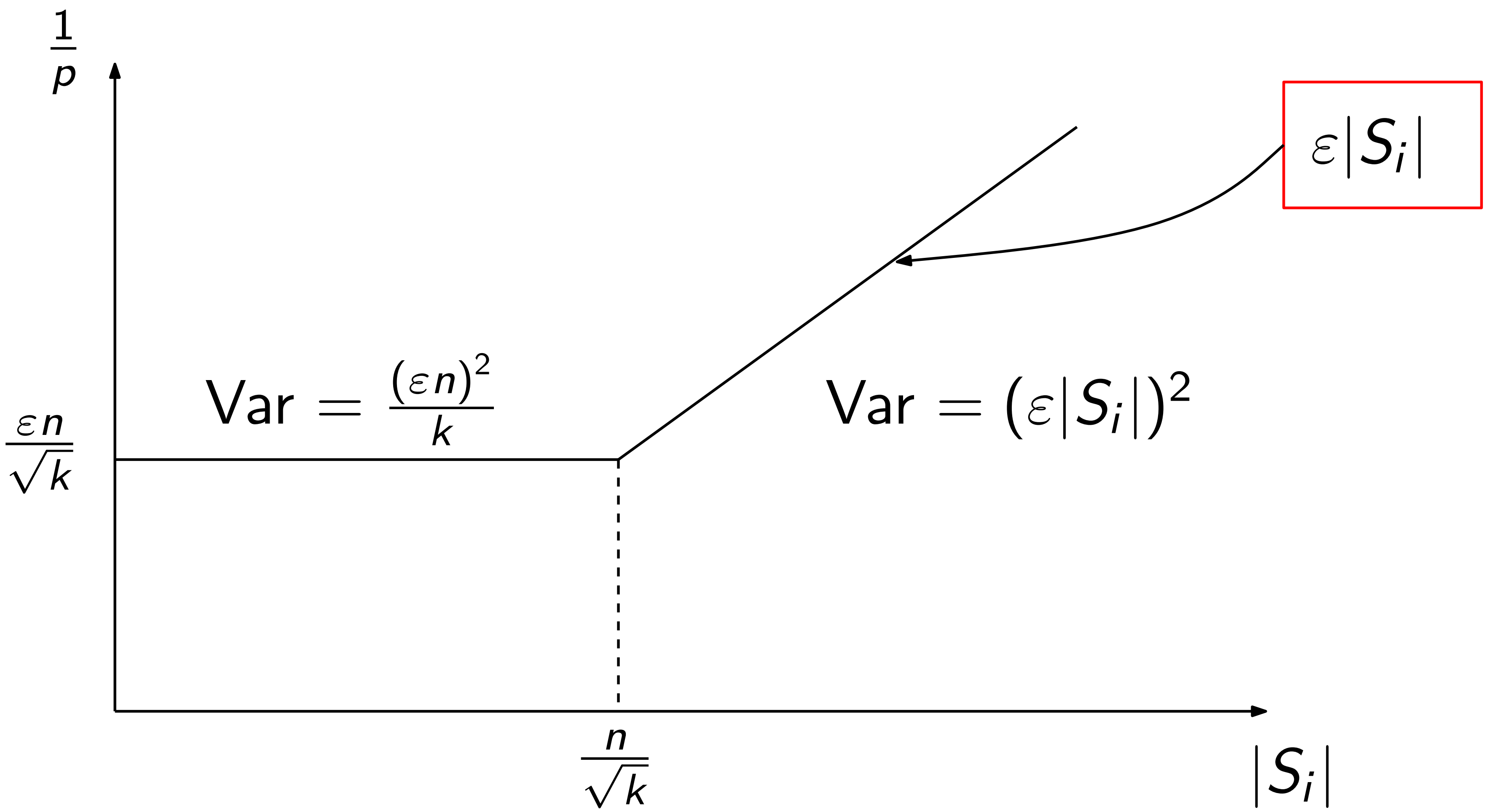

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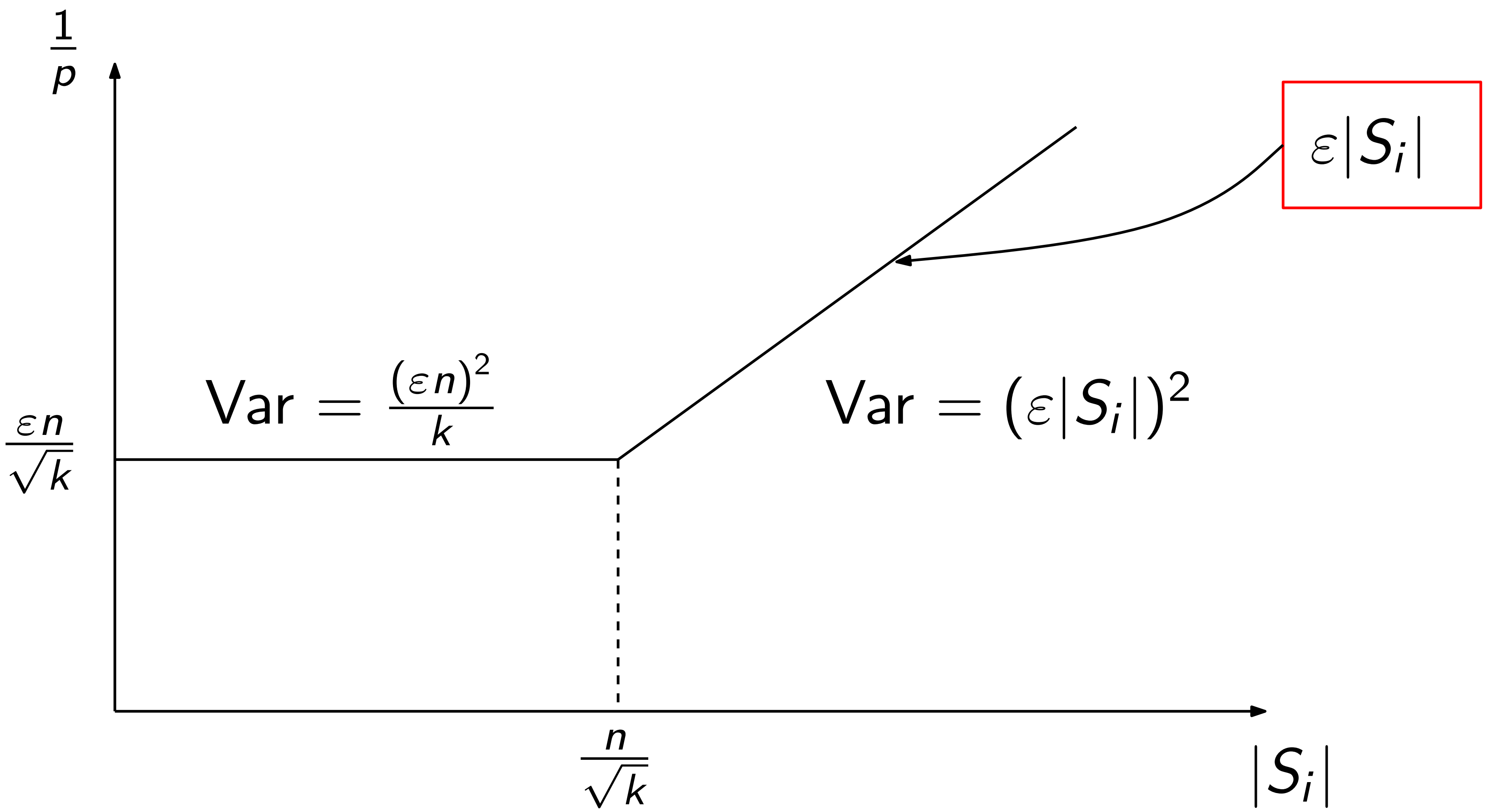
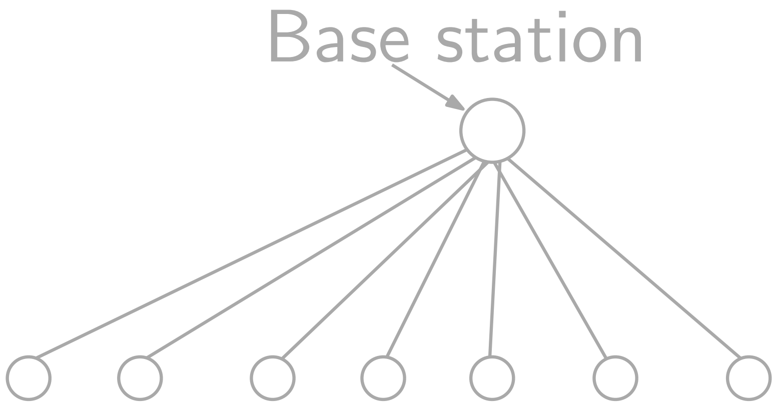


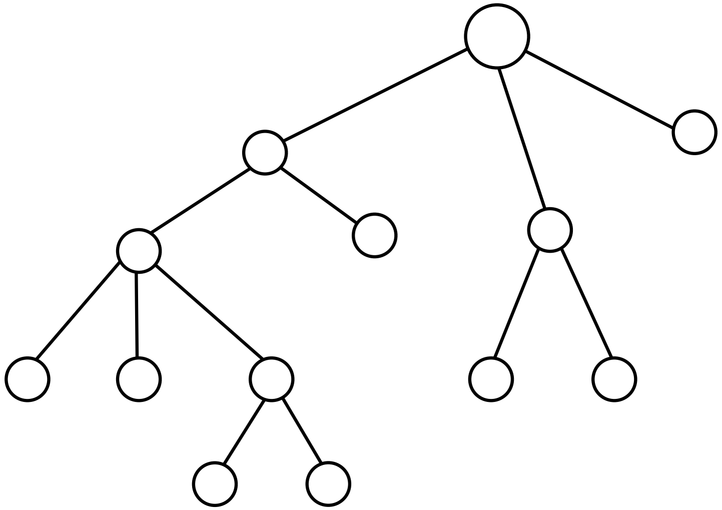
$$\text{Total variance} \leq \sum_i \left(\frac{(\varepsilon n)^2}{k} + (\varepsilon |S_i|)^2\right) \leq 2(\varepsilon n)^2$$

Roadmap



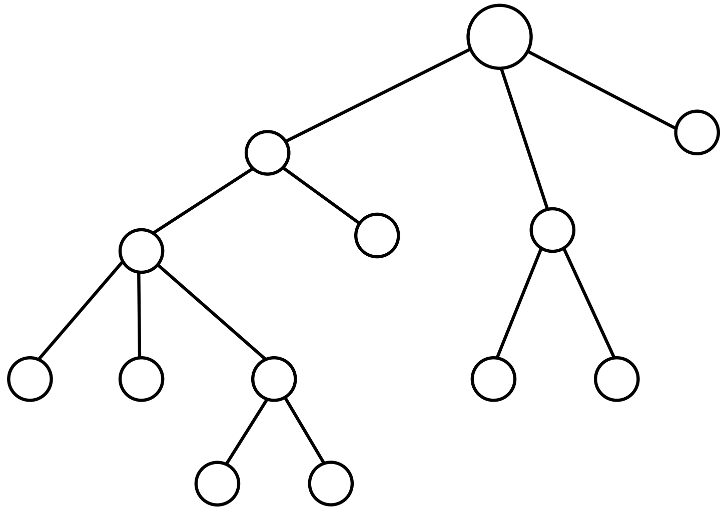
- Flat model

- Tree model



Tree Model: Naive Extension from Flat Model

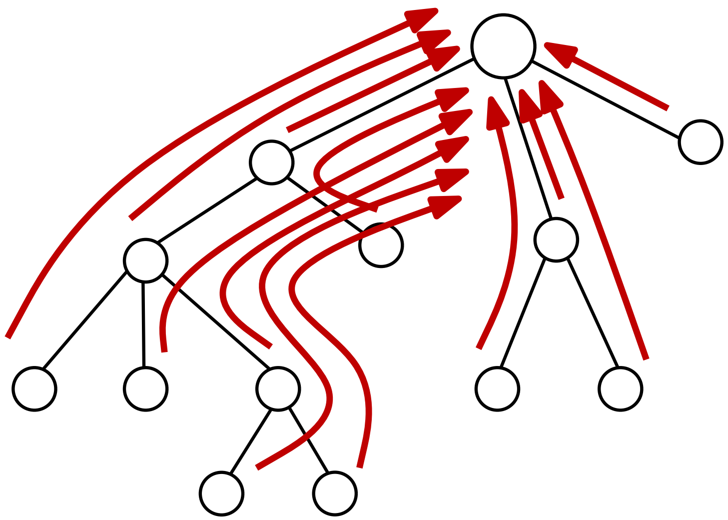
Total cost: $\frac{\sqrt{k}}{\varepsilon}h$ (h is the height of the tree)



Tree Model: Naive Extension from Flat Model

Total cost: $\frac{\sqrt{k}}{\varepsilon}h$ (h is the height of the tree)

Max individual cost: $O(\frac{\sqrt{k}}{\varepsilon})$



Tree Model: Merging Samples

S_i ① ③ ④ ⑥ ⑦ ⑨ ⑪ ⑬ ⑯ ㉖ ㉑ ㉔

p_i $(3,2)$ $(7,5)$ $(13,8)$ $(26,10)$

$$p_i = \begin{cases} \frac{\sqrt{k}}{\varepsilon n}, & |S_i| \le \frac{n}{\sqrt{k}} \\ \frac{1}{\varepsilon |S_i|}, & |S_i| \ge \frac{n}{\sqrt{k}} \end{cases}$$

Tree Model: Merging Samples

S_i 1 3 4 6 7 9 11 13 16 26 21 24

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$|S_1|$ p_1 $(3,2)$ $(7,5)$ $(13,8)$ $(26,10)$

$+$

$|S_2|$ p_2 $(5,3)$ $(14,6)$ $(18,8)$ $(24,11)$

$=$

$|S_1| + |S_2|$ p

$\left(> \frac{n}{\sqrt{k}}\right)$

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$|S_1|$ p_1 $(3,2)$ $(7,5)$ $(13,8)$ $(26,10)$

Sample w.p. p/p_1

$+$

$|S_2|$ p_2 $(5,3)$ $(14,6)$ $(18,8)$ $(24,11)$

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Sample w.p. p/p_2

$|S_1| + |S_2|$ p

$\left(> \frac{n}{\sqrt{k}}\right)$

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$|S_1| + |S_2|$ p $(13, 8+?)$

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Sample w.p. p/p_1

Sample w.p. p/p_2

$|S_1| + |S_2|$ p $(13, 8+?)$

$\left(> \frac{n}{\sqrt{k}}\right)$

This is a rank query in the other sample!

Tree Model: Merging Samples

Problem: Variances accumulate

(*Law of total variance*)

$$|S_1| \quad p_1 \qquad (3,2)\ (7,5)\ (13,8)\ (26,10)$$

$$+$$

$$|S_2| \quad p_2 \qquad (5,3)\ (14,6)\ (18,8)\ (24,11)$$

$$=$$

$$(13, 8+3+1/p_2)$$

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$$=$$

$$(13, 8+3+1/p_2)$$

Var[local count of 13 in $S_1 \cup S_2$] $=$
Var[local count of 13 in S_1] $+$
Var[local count of 5 in S_2] $+$
$1/p_2^2$

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Problem: Variances accumulate

(*Law of total variance*)

$|S_1| \quad p_1 \qquad (3,2)\ (7,5)\ (13,8)\ (26,10)$

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$=$

$(13, 8+3+1/p_2)$

Var[local count of 13 in $S_1 \cup S_2$] $=$
Var[local count of 13 in S_1] $+$
Var[local count of 5 in S_2] $+$
$1/p_2^2$

Var[merged sample] $=$
Var[sample of S_1] $+$
Var[sample of S_2] $+$
$\max(1/p_1^2, 1/p_2^2)$

Tree Model: Merging Samples

Problem: Variances accumulate

(*Law of total variance*)

$$|S_1| \quad p_1 \qquad (3,2)\ (7,5)\ (13,8)\ (26,10)$$

$$+$$

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$$=$$

$$(13, 8+3+1/p_2)$$

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$1/p_2^2$

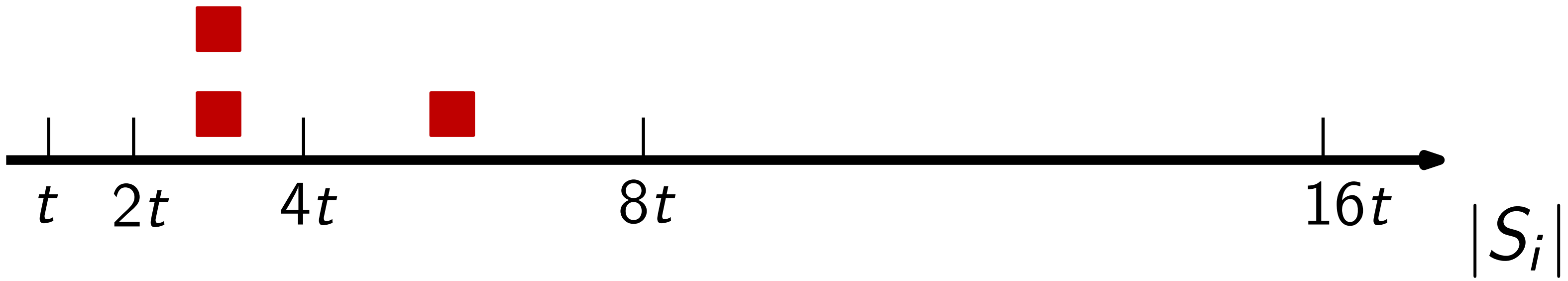
Var[merged sample] $=$
Var[sample of S_1] $+$
Var[sample of S_2] $+$
$\max(1/p_1^2, 1/p_2^2)$

Get "penalized" if we merge two samples of uneven sizes.

Tree Model: Merging Samples

Idea: Only merge samples with $p_1 \approx p_2$, i.e., $|S_1| \approx |S_2|$

$t = \frac{n}{\sqrt{k}}$



Merge samples in the same bucket

Tree Model: Merging Samples

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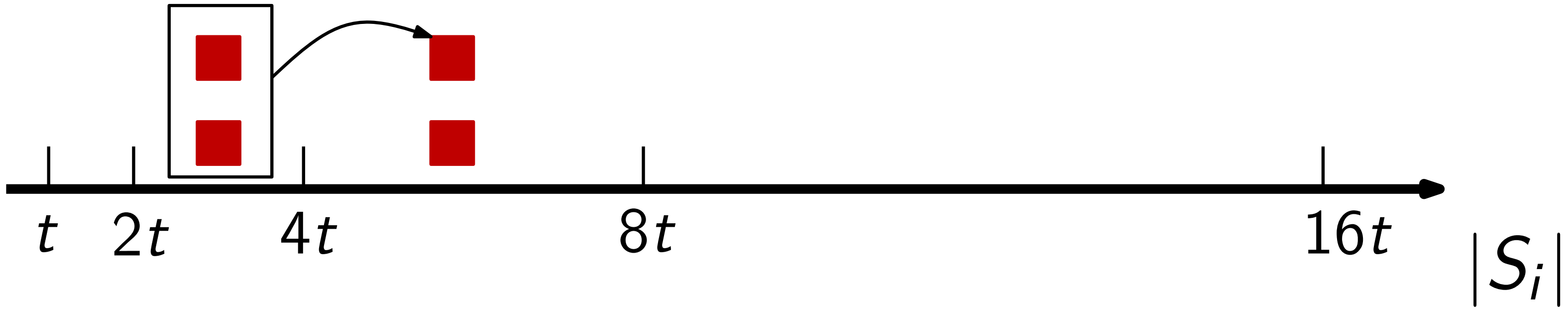

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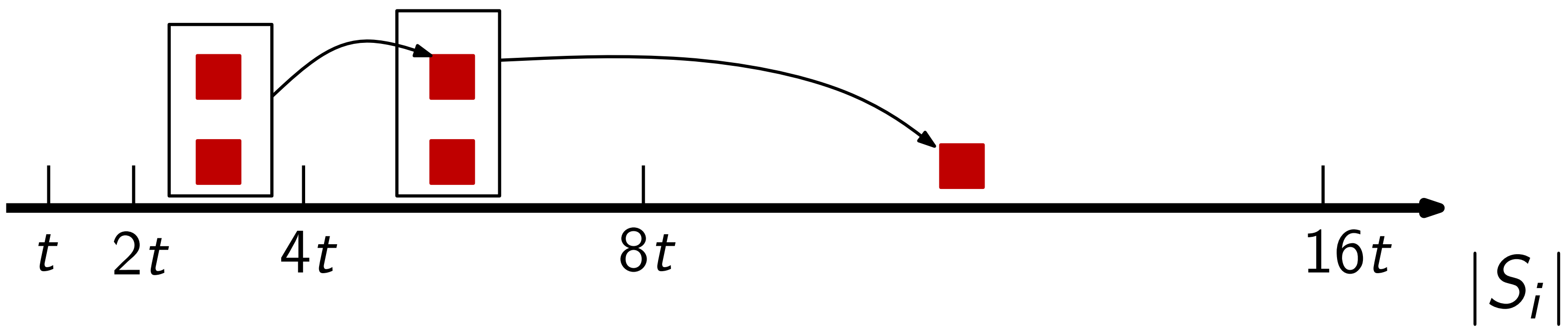

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(so, the max individual cost is $O(\frac{1}{\varepsilon} \log k)$)

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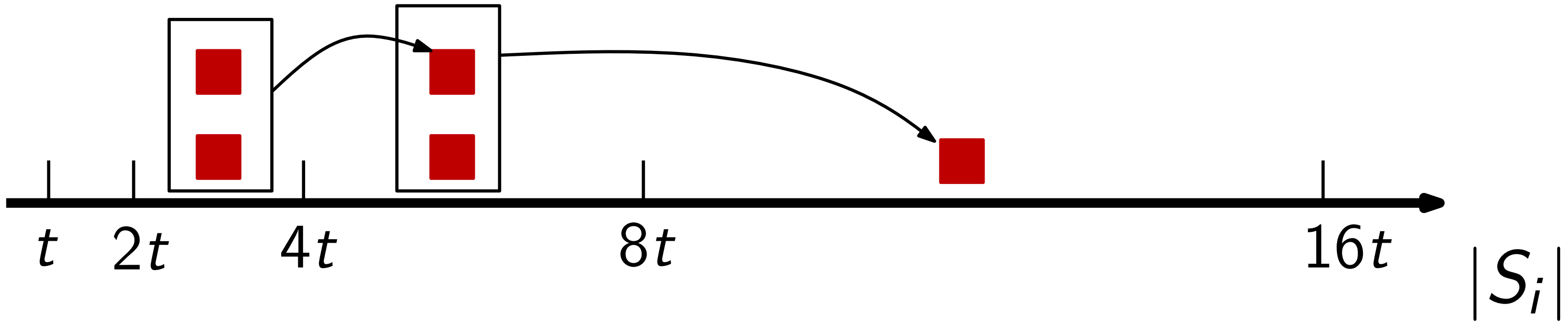

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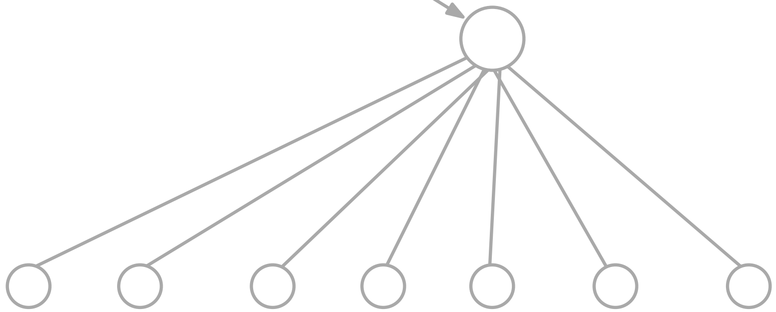
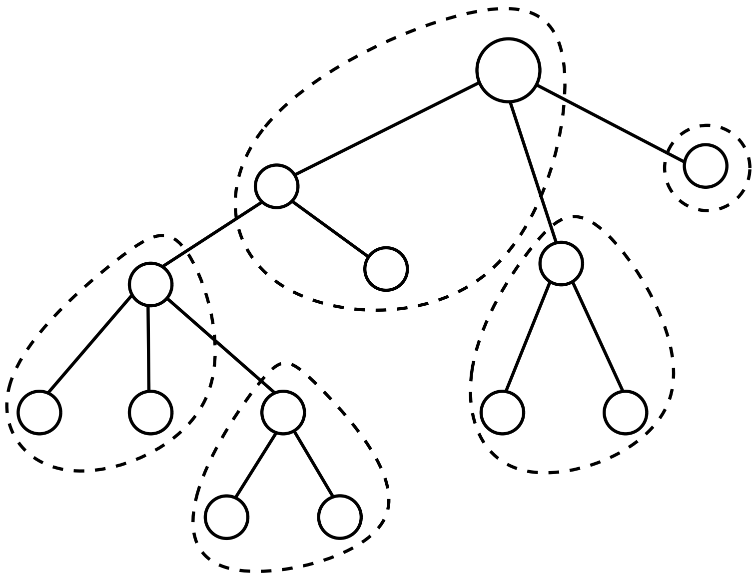
Can show that the final variance is $O((\varepsilon n)^2)$ (please see paper)

Roadmap



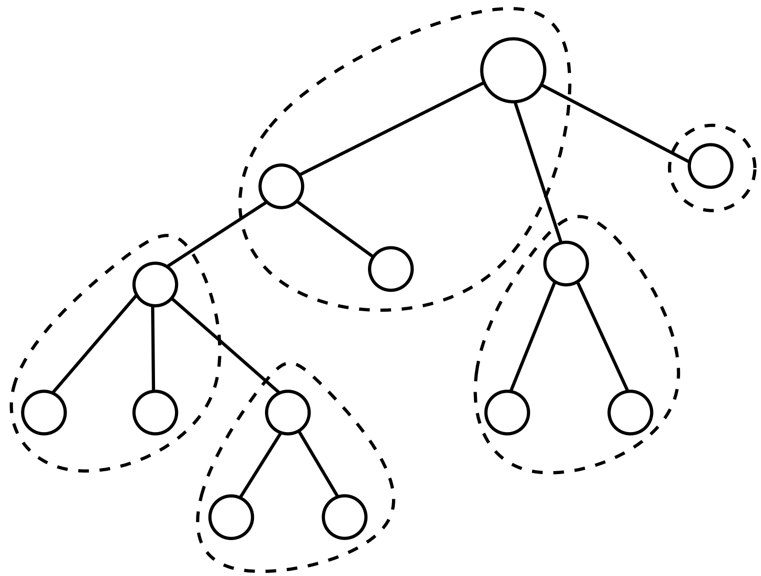
- Flat model
- Tree model



- Partitioned tree

Tree Partitioning

Partition into t connected components with $O(k/t)$ nodes each



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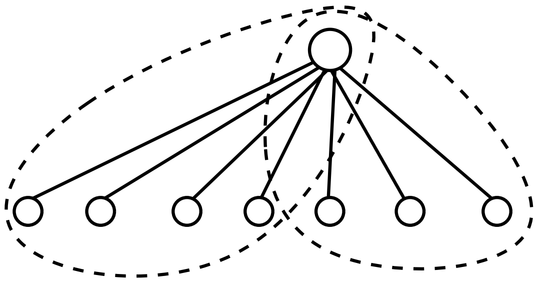
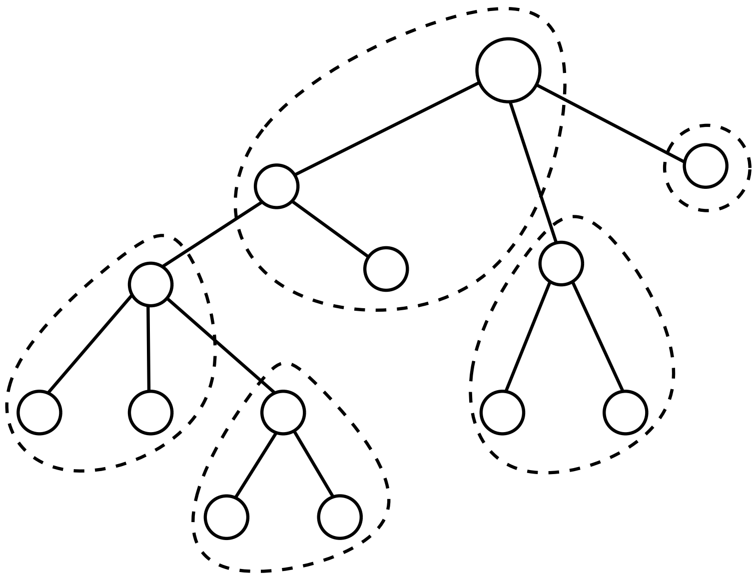
May need to share a few ($\leq t$) nodes





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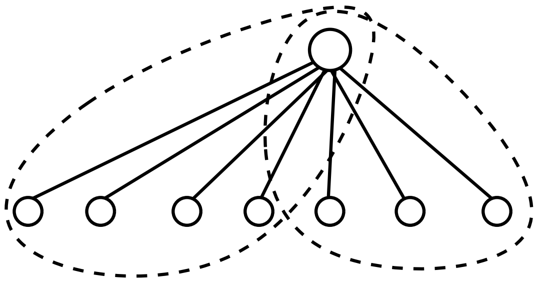
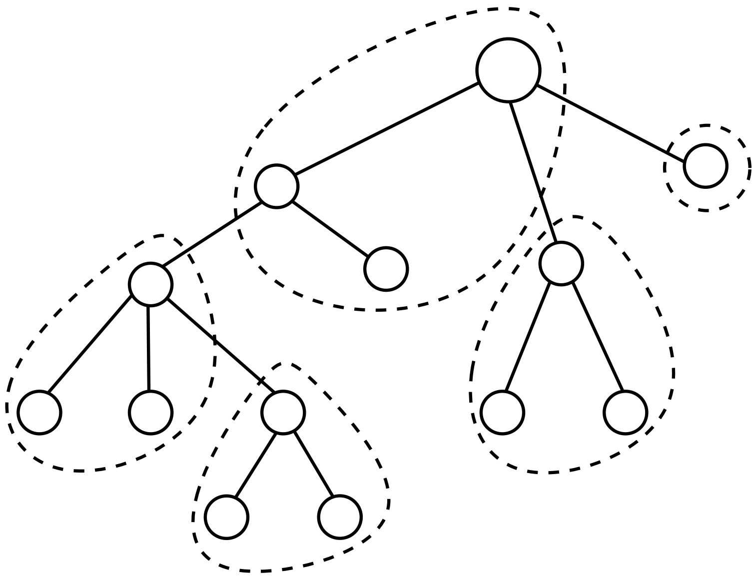
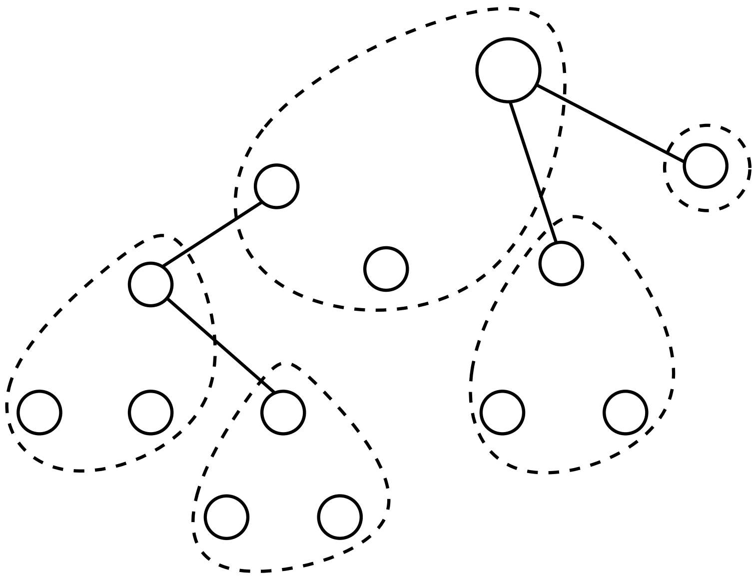




Can be done in linear time and communication.
Algorithm not too difficult
— a nice homework question for an algorithms course?

Quantile Computation on a Partitioned Tree

Consider the t components as t “super nodes”
Total sampe size is $O(\sqrt{t}/\varepsilon)$, communication cost $O(\sqrt{t}/\varepsilon \cdot h)$



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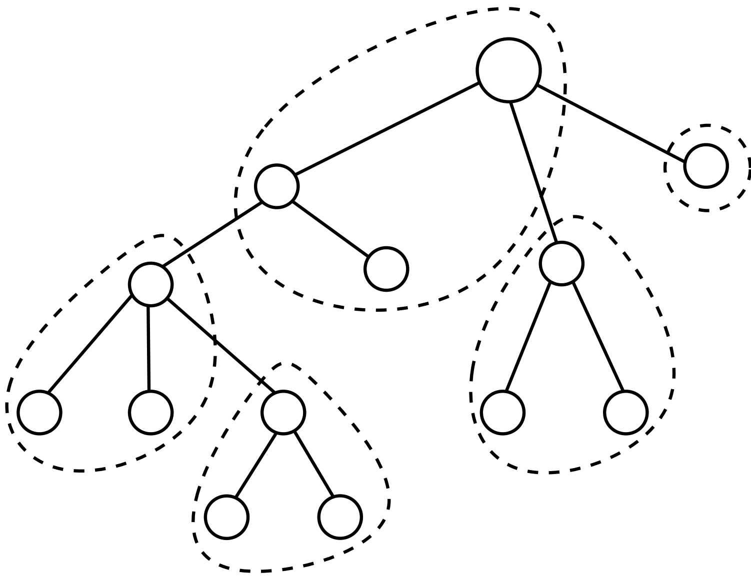

Computing the "local ranks" of the sampled values, need to broadcast them to all nodes in the super node, costing $O(\sqrt{t}/\varepsilon \cdot k/t)$

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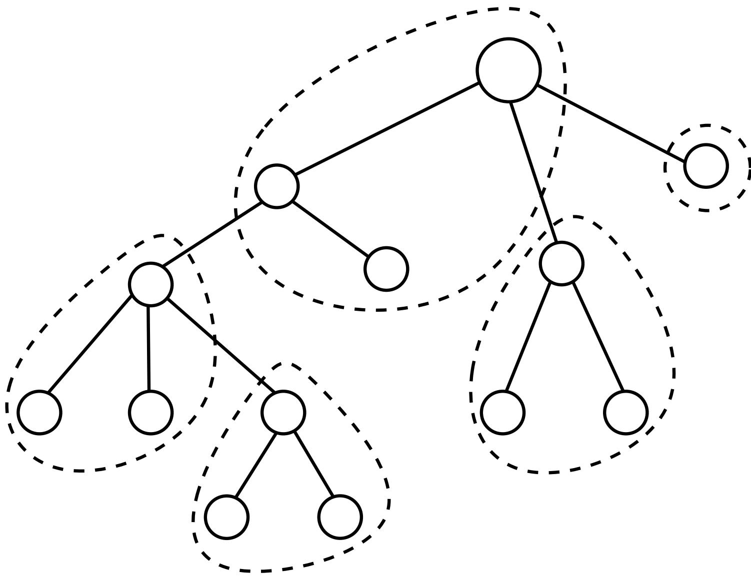
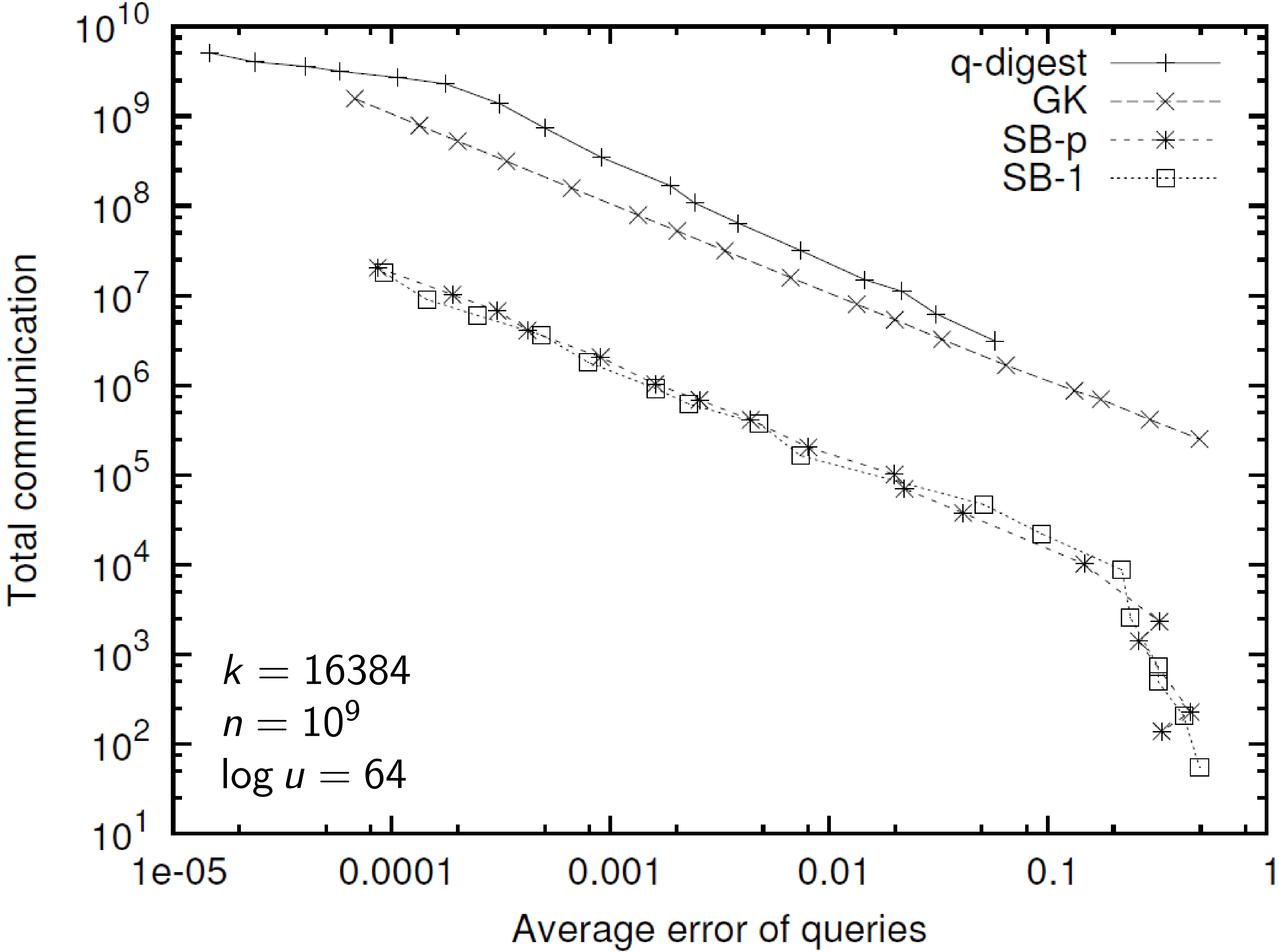


Need $h = k/t$ to balance

When $t = k/h$, both are $O(\sqrt{kh}/\varepsilon)$

Computing the "local ranks" of the sampled values, need to broadcast them to all nodes in the super node, costing $O(\sqrt{t}/\varepsilon \cdot k/t)$

Experimental Results on Terrain Data



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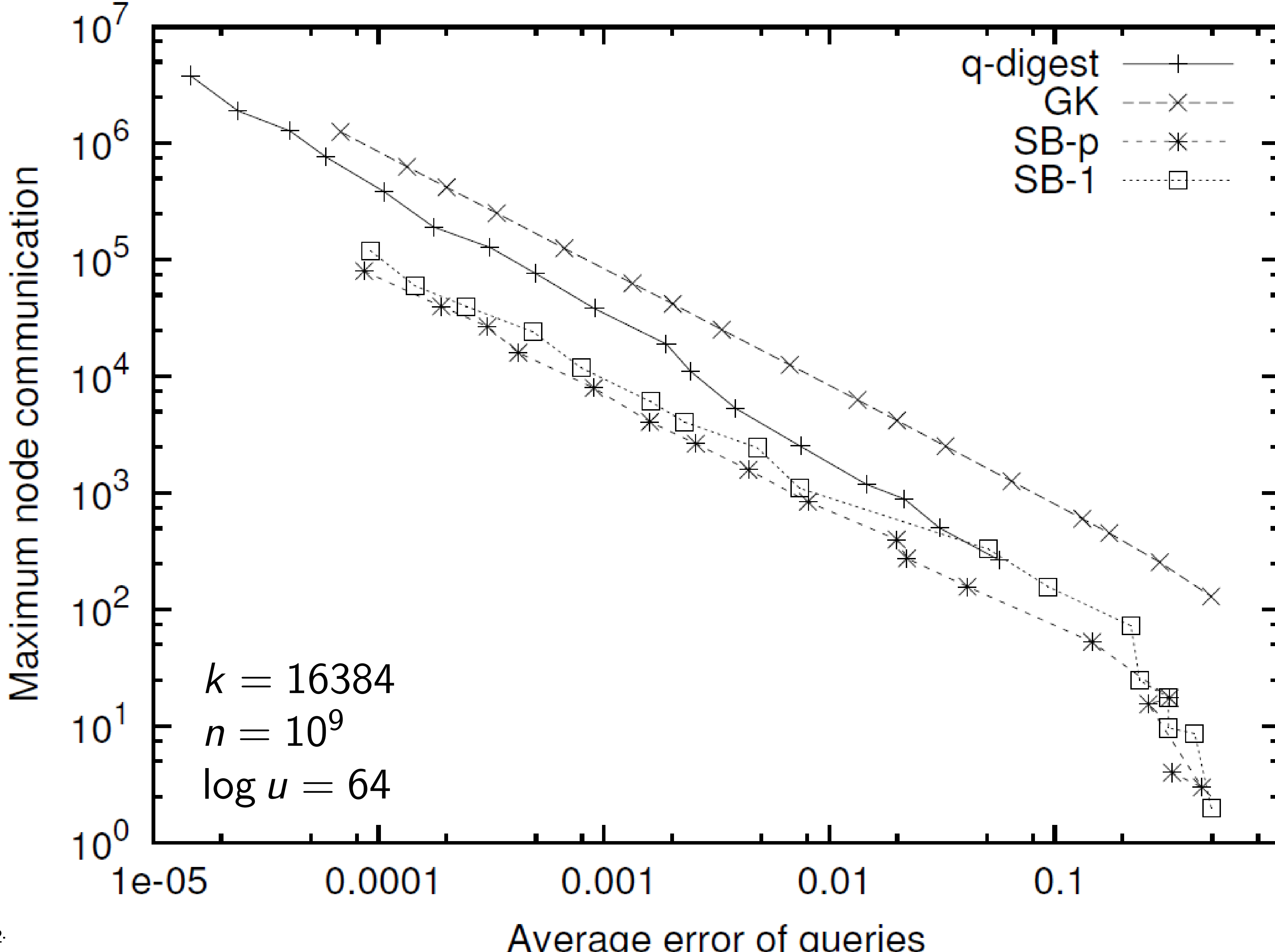

Conclusion

- A sampling based algorithm whose total communication cost grows sublinearly as network size
- Deviate from the traditional "decomposable" framework
- Can we solve other data aggregation problems using similar ideas?