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Algebraic Relations among Matching Constraints of Multiple Images

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Thème 3 — Interaction homme-machine,
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Projet MOVI

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Abstract: Given a set of $n \geq 2$ uncalibrated views, for any corresponding point across n views, there exist three types of matching constraints: bilinear constraints (for $n \geq 2$), trilinear constraints (for $n \geq 3$, [12]) and quadrilinear constraints (for $n \geq 4$, [6, 14, 3]). The exact algebraic relations among these multi-linear constraints have not been elucidated by previous authors. This paper examines the relations between these matching constraints by singling out the degenerate view and point configurations. The key result that will be established is that for generic view configurations and generic points, all multi-linear constraints may algebraically be reduced to the algebraically independent bilinear constraints. In other words, all matching constraints are contained in the ideal generated only by the bilinear constraints for generic views and points. As a consequence, $2n-3$ algebraically independent bilinearities from pairs of views completely describe the algebraic/geometric structure of n uncalibrated views for generic views and points. For degenerate points of generic views, each type of constraint reduces differently. The exact reduced form of the matching constraints are also made explicit by computer algebra.

Key-words: geometry, invariant, epipolar geometry, bilinearity, trilinear ity, uncalibrated image.

(Résumé : tsvp)

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Relations algébriques entre les contraintes géométriques d'images multiples

Résumé : Il est connu qu'il existe 3 types de contraintes géométriques pour les points en correspondence dans les images multiples: relations bilinéaires, trilinéaires et quadrilinéaires. Nous démontrons que toutes les relations multilinéaires se re'duisent en relations bilinéaires pour les configurations générales des points et des caméras. Les formes réduites et exactes des relations trilinéaires sont aussi exhibées par le calcul symbolique pour les points en configurations dégénérées et les caméras en configurations générales.

Mots-clé : géométrie, géométrie épipolaire, bilinéarité, trilinéarité, caméra non-calibrée

1 Introduction

Recently, a number of works have been concentrated on the analysis of multiple uncalibrated images [2, 7, 10, 11]. Thanks to the pioneering work of Shashua [12], Hartley [6], Triggs [14], Faugeras and Mourrain [3, 4], Luong and Vieville [9, 15] and Werman and Shashua [16] on the study of the geometric relationships between different uncalibrated views, it has been shown that there are only three types of algebraic relations on image points between any uncalibrated views: Bilinearities for two views, trilinearities for three views and quadrilinearities for four or more. It has also been well established that quadrilinearities are not independent; they can be generated from (independent) trilinearities and bilinearities [14, 3] and further more bilinearities can be generated by trilinearities. However it is unclear if trilinearities can be generated by (independent) bilinearities. In [14], by geometric arguments, it is suggested that the trilinearities are generated by the bilinearities. In this paper, the exact relations among those multi-linear constraints will be studied. One key idea is to introduce the generic/degenerate view configurations and point configurations and make distinctions between these two different kinds of degenerate configurations. This allows us to establish that all trilinearities follow the bilinearities for generic point and view configurations. Therefore, it can be concluded that all multi-linear constraints may be reduced to the independent bilinearities. For degenerate points, all constraints are reduced, and we will give the exact reduced forms of the matching constraints.

This paper is organised as follows. Section 2 reviews the derivations of multi-linear constraints. Then the case of three views will be studied in detail in Section 3 in which some major results will also be presented. After that, Section 4 basically extends the resuls of three views to the general n view case. Finally, some concluding remarks are given in Section 5.

2 Derivation of multi-linear matching constraints: review

Following the general formalism proposed by Faugeras [3] and Triggs [14] for deriving the algebraic relations among views, the multi-linear constraints may be derived as follows.

For each view, consider a point $\mathbf{x}^T = (x, y, z, t)^T$ in $\mathcal{P}^3$ projected onto a point $\mathbf{u}^T = (u, v, w)^T$ in $\mathcal{P}^2$ by a 3×4 matrix $\mathbf{P}_{3\times 4} = (p_{ij})$ as

$$\lambda(u, v, w)^T = P_{3\times 4}(x, y, z, t)^T, \tag{1}$$

for short, $\mathbf{u}^T = P_{3\times 4}\mathbf{x}^T$. Then equation (1) can be rewritten in terms of $\mathbf{x}$ as

$$\begin{pmatrix} u\mathbf{p}_3^T - w\mathbf{p}_1^T \\ v\mathbf{p}_3^T - w\mathbf{p}_2^T \end{pmatrix} \mathbf{x} = 0,$$

where $(\mathbf{p}_1^T\ \mathbf{p}_2^T\ \mathbf{p}_3^T)^T = P_{3\times 4}$.