The Hong Kong University of Science & Technology
Department of Computer Science

COMP 171: Data Structures and Algorithms
Fall 2005

Written Assignment 2
Out on November 11, 2005
Due: Tuesday, November 29, 2005

Your answers will be graded on clarity, correctness, efficiency, and precision.

1. You are given a binary tree T with $n \geq 1$ nodes. Each node has a parent pointer (null for the root) and two child pointers, either of which may be null. Each node x has an integer key $K(x)$. No two keys in T are equal. For each task below, describe an efficient algorithm and derive the worst-case time complexity of your algorithm.

   (a) Determine if T is a binary search tree.

   (b) Determine if T is an AVL tree.

2. Write a routine to list out the nodes of a binary tree in *level-order*. List the root, then nodes at depth 1, followed by nodes at depth 2, and so on. You must do this in linear time. Prove the time bound of your algorithm.

3. (a) Insert the following sequence of numbers in this order into an initially empty B^+-tree with $M = 5$. Use the insertion procedure described in class. Draw the B^+-tree after the insertion of each number.

   ```
   6 2 4 7 10 11 13 15 5 3 12 14 9 8 16 1 17 18
   ```

   (b) Delete the following sequence of numbers in this order from the final B^+-tree in part (a). Use the deletion procedure described in class. Draw the B^+-tree after the deletion of each number.

   ```
   5 6 7 8 10 9
   ```

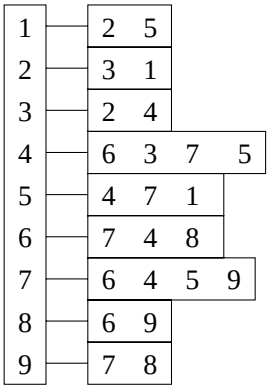
4. You are given a binary search tree T with $n \geq 1$ nodes. Each node has a parent pointer (null for the root) and two child pointers, either of which may be null. T might not be balanced. Let h denote the height of T. Assume that you have a pointer to a node x in T. Define the *first successor* of x as the successor of x. For $k \geq 2$, we define the *kth successor* of x to be the successor of the $(k-1)$th successor of x. For $k \geq 1$, if the kth successor of x does not exist, it is defined as null.

   (a) Write a procedure to find the first successor of x. Explain, and illustrate with figures, how you handle the various possible scenarios.

   (b) What is the worst-case time complexity of finding the first successor of x? Explain your answer.

   (c) For an arbitrary k greater than 1, one can find the kth successor of x by invoking the successor function k times in a row. What is the worst-time complexity of doing this? Explain your answer. (Hint: an easy answer is to multiply your answer in (a) by k, but there is a better one.)

5. You are given an undirected graph with the following adjacency list representation.



   (a) Draw the BFS tree obtained by running BFS from the vertex 3.

   (b) Draw the DFS tree obtained by running DFS from the vertex 3.

6. The objective of the *Kevin Bacon Game* is to link a movie actor to the actor Kevin Bacon via shared movie roles. An actor's Bacon number is defined as the minimum number of links from Kevin Bacon. For instance, Tom Hanks has a Bacon number of 1, because he was in *Apollo 12* with Kevin Bacon. Sally Fields has a Bacon number of 2, because she was in *Forest Gump* with Tom Hanks, who was in *Apollo 13* with Kevin Bacon. Almost all well-known actors have a Bacon number of 1 or 2. Assume that you have a comprehensive list of actors, with their movie roles.

   (a) Explain how to find an actor's Bacon number.

   (b) Explain how to find the actor who has the highest Bacon number.

   (c) Explain how to find the minimum number of links between two arbitrary actors.