

The Hong Kong University of Science & Technology  
Department of Computer Science

COMP 171: Data Structures and Algorithms  
Written Assignment 1

Out on October 5, 2005

Due on October 18, 2005 (at the beginning of class)

Your answers will be graded on clarity, correctness, efficiency, and precision.

- For each pair of  $f(n)$  and  $g(n)$  below, decide if  $f(n) = O(g(n))$ ,  $f(n) = \Omega(g(n))$ , or  $f(n) = \Theta(g(n))$ . Justify your answer using the definitions of these asymptotic notation. Note that more than one of these relations may hold for a given pair; list all correct ones.
  - $f(n) = \sqrt{n}$  and  $g(n) = \log_2 n$ .
  - $f(n) = \log_2^3 n$  and  $g(n) = \log_2 n^3$ .
  - $f(n) = 2^n$  and  $g(n) = 2^{n/2}$ .
  - $f(n) = \log_2(n!)$  and  $g(n) = n \log_2 n$ .
- Let  $f(n)$  and  $g(n)$  be asymptotically positive functions. Prove or disprove each of the following conjectures.
  - $f(n) = O(g(n))$  implies  $g(n) = O(f(n))$ .
  - $f(n) = \Theta(f(n/2))$ .
- Solve the following recurrence relation:  $T(1) = 1$ ,  $T(n) = T(\frac{n}{2}) + O(n)$ , where  $n > 1$ .
- Describe an algorithm to perform mergesort non-recursively. Use plain English text. Do not give us any C++ code or pseudo-code.
- Is the array  $\{23, 17, 14, 6, 13, 10, 1, 5, 7, 12\}$  a heap?
  - Given  $k$  sorted lists containing a total of  $n$  elements. Design an  $O(n \log k)$  algorithm to merge these lists into a single sorted list.
- Let  $A[0..n - 1]$  be a (min) heap of size  $n$ . Let  $A[j]$  be a specific entry given to you. You are to design algorithms to support the following operation on the heap. Given an input parameter  $k$  such that  $k > A[j]$ , describe an algorithm to increase the value of  $A[j]$  to  $k$  using plain English text. (Do not give us any C++ code or pseudo-code.) Note that  $j$  might not be equal to 0 or  $n - 1$ . Your algorithm should restore the heap order after increasing the value of  $A[j]$ . Analyze the worst-case running time of your algorithm. Your algorithm should be as efficient as possible. In particular, rebuilding a heap from scratch is not a satisfactory solution.