

The Hong Kong University of Science & Technology
Department of Computer Science

COMP 171: Data Structures and Algorithms
Written Assignment 1

Out on October 5, 2005

Due on October 18, 2005 (at the beginning of class)

Your answers will be graded on clarity, correctness, efficiency, and precision.

- For each pair of $f(n)$ and $g(n)$ below, decide if $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. Justify your answer using the definitions of these asymptotic notation. Note that more than one of these relations may hold for a given pair; list all correct ones.
 - $f(n) = \sqrt{n}$ and $g(n) = \log_2 n$.
 - $f(n) = \log_2^3 n$ and $g(n) = \log_2 n^3$.
 - $f(n) = 2^n$ and $g(n) = 2^{n/2}$.
 - $f(n) = \log_2(n!)$ and $g(n) = n \log_2 n$.
- Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.
 - $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.
 - $f(n) = \Theta(f(n/2))$.
- Solve the following recurrence relation: $T(1) = 1$, $T(n) = T(\frac{n}{2}) + O(n)$, where $n > 1$.
- Describe an algorithm to perform mergesort non-recursively. Use plain English text. Do not give us any C++ code or pseudo-code.
- Is the array $\{23, 17, 14, 6, 13, 10, 1, 5, 7, 12\}$ a heap?
 - Given k sorted lists containing a total of n elements. Design an $O(n \log k)$ algorithm to merge these lists into a single sorted list.
- Let $A[0..n-1]$ be a (min) heap of size n . Let $A[j]$ be a specific entry given to you. You are to design algorithms to support the following operation on the heap. Given an input parameter k such that $k > A[j]$, describe an algorithm to increase the value of $A[j]$ to k using plain English text. (Do not give us any C++ code or pseudo-code.) Note that j might not be equal to 0 or $n-1$. Your algorithm should restore the heap order after increasing the value of $A[j]$. Analyze the worst-case running time of your algorithm. Your algorithm should be as efficient as possible. In particular, rebuilding a heap from scratch is not a satisfactory solution.