Approximation Algorithms – COMP 670J
Fall Semester 2001
Homework 4 – Due December 12, 2001

Instructions: Answer all of the following questions. Please *fully* prove your solutions. Follow the rules concerning collaboration and plagiarism as laid out in the first class handout. Write your name, ID number, and email address clearly on your answer sheet and hand in the homework by 2:00PM on Wednesday December 12, 2001 to either the instructor or Ms. Connie Lau in the CS department administration office. Please *typeset* or type all of your solutions (included figures can be hand-drawn if necessary).

1. *Complementary slackness* (4 pts) Recall the (relaxed) LP for set-cover given on page 5 of the *primal-dual* lecture notes (this is the same as LP (13.2) on page 109 in the Vazirani book).

Minimize $\sum_{F \in \mathcal{F}} C(F) x_F$	
subject to conditions	
$\forall e \in U,$	$\sum_{e \in F} x_F \geq 1$
$\forall F \in \mathcal{F},$	$x_F \geq 0$

   Consider the following algorithm: exactly solve the relaxed LP to get an optimal solution with values $x_F \geq 0$. Now set $C = \{F : x_F > 0\}$.

   - Show that C is a set cover.
   - Use complementary slackness or relaxed complimentary slackness to show that C is within factor f of the optimal cover (where f is the maximum frequency of an element as defined on page 5 of the primal-dual slides).

2. *Multicut I* (2 pts) Suppose that lines 3 and 4 (deletion of edges from D) are removed from the multicut algorithm on page 14 of the primal-dual lecture notes. Prove that the approximation factor of the resulting algorithm is unbounded.

3. *Multicut II* (3 pts) Prove that if e and e' are both in D in step 3 of the multicut algorithm on page 14 of the primal-dual lecture notes and e is deeper than e', then e is added before or in the same iteration as e'.

   (This is question 18.8 on page 153 of the Vazirani book.)

4. *Multicut III* (2 pts) Do Exercise 18.9 on page 153 of the Vazirani book.