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12.1-2

Recall the following:

- Binary-search-tree property: Let *x* be a node in a binary search tree. If *y* is a node in the left subtree of *x*, then key(*y*) ≤ key(*x*). If *y* is a node in the right subtree of *x*, then key(*x*) ≤ key(*y*).
- Min-heap property: For every node *i* other than the root, A[Parent(*i*)] ≤ A[i]

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a) The difference

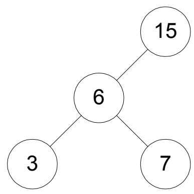
Let x be a node. A heap requires that key(x) must be less than or equal to both key[left[x]] and key[right[x[]]. But a binary tree requires that key[x] ≥ key[left[x]], i.e. key[x] cannot be less than key[left[x]].

b)

The min-heap property cannot be used to print out the keys in sorted order in O(n) time.
We only know that a node must be less than or equal to its children. Assume that we can compare in constant time two children y, z, and decide which subtree to traverse first. But we still cannot know if children of y are greater than children of z, or vice versa. So there is no rule stating the relationship between children. Thus, we cannot use an inorder tree walk to print the keys in sorted order in O(n) time. Linear sorts such as radix sort only work on integers. Therefore, in general we cannot use heap property to print out the keys in sorted order in O(n) time.

12.2-4

A counter-example: If we search 3 from the following tree, then A = {}, B={3,6,15}, C={7}.
Let b=15 and c=7. But b > c (contradictory).



12.2-7

[The key is to show that each node will be visited at most 3 times]
Let *x* be a node. Consider the following case:

Case 1: *x* must be on the simple path from the root of a subtree to the local minimum, because *x* will be visited by the first TREE-MINIMUM or the TREE-MINIMUM in TREE-SUCCESSOR.

Case 2:
If *x* has a left subtree *L*, then the minimum *m* of the subtree rooted at *x* is in *L*. The algorithm will return *m*, then the successor of *m*, and so on until the predecessor *p* of *x* is returned. When Tree-Successor(*p*) is called, *x* will be visited and returned.

Case 3:
If *x* has a right subtree, then the successor of *x* is in the right subtree. Then the inorder walk will return all nodes in the right subtree. After that, TREE-SUCCESSOR will visit *x* again in order to find the next successor (which may be parent(x)).

As a result, node *x* will be visited at least once and most 3 times by the algorithm.

13.1-5

By lemma 13.1, a red-black tree with *n* internal nodes has height at most 2lg(n+1). Thus, the longest simple path from a node *x* to a descendant leaf is at most 2lg(n+1).

Let *h* be the height of the tree. According to red-black tree property 3 – if a node is red, then both its children are black, at least half of the nodes on any simple path from the root to a leaf, not including the root, must be black.

Thus, $h/2 \le bh(x)$, where $h \le 2\lg(n+1) \Rightarrow bh(x) \ge \lg(n+1)$ = the shortest path is the path with only black nodes. Therefore, the longest simple path is at most twice that of the shortest simple path.

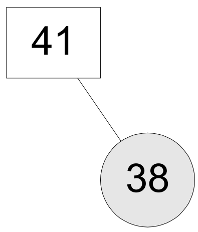
13.3-2

(Sentinels will not be shown)

#1

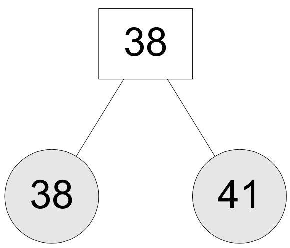
A square represents a black node

#2



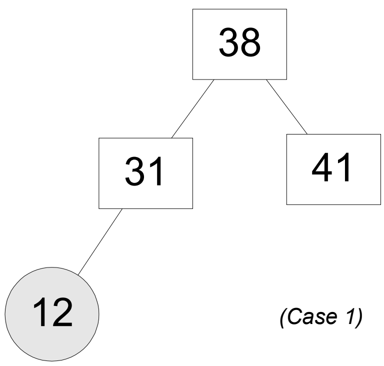
A circle represents a red node

#3



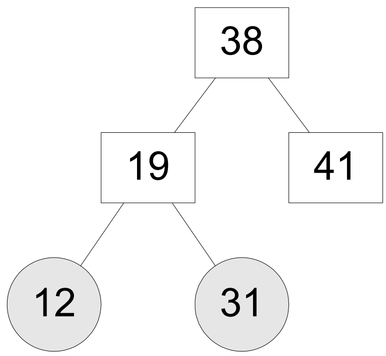
(Case 3)

#4



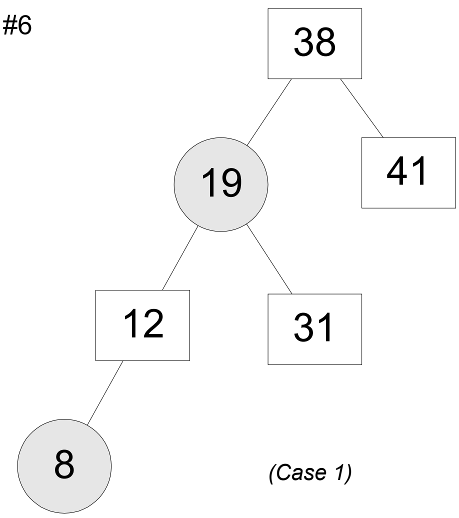
(Case 1)

#5



(Case 2, then Case 3)

#6



(Case 1)

13.4-7

The resulting red-black tree is not always the same. Consider the following case: