TSP with Release Times and Distance Constraints

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1 TSP with release times

Given a complete graph $G = (V, E)$ with vertex set $V = \{0, 1, 2, \ldots, n\}$ and edge set E, there is a salesman, initially located at vertex 0(called its home), who wants to visit each vertex in $V \setminus \{0\}$ to sale his goods. Moreover, each vertex i can be visited no earlier than its release time r_i and the travel time of edge (i, j) is $t_{i,j}$, where $t_{(\cdot,\cdot)}$ is a metric function on the edge set E. Our goal is to find a schedule S or a visiting order of the vertices for the salesman to minimize the makespan, i.e. the time elapsed when visiting all the vertices and come back home.

Previous results. Nagamochi et al. [2] gave a 5/2-approximation algorithm in which the salesman first waits at vertex 0 until the largest release time r_n and then travels along an approximate tour T generated by Christofides' Algorithm.

An interesting problem is to devise an approximation algorithm with performance ratio less than 5/2.

2 Distance constrained vehicle routing problems

Given a complete graph $G = (V, E)$ with vertex set $V = \{0, 1, 2, \ldots, n\}$ and edge set E, the length of edge (i, j) is $t_{i,j}$, where $t_{(\cdot,\cdot)}$ is a metric function on the edge set E. A rooted tour is a tour on some subset of V that includes the root vertex 0. Also given is a distance bound D. The problem is to find a minimum number of rooted tours(each of length at most D) to cover each vertex. We denoted this problem by DVRP.

Previous results. Nagarajan et al. [3] gave a $\min\{\log n, \log D\}$-approximation algorithm for DVRP. However, for unrooted version of the problem(in which there is no root and hence each vertex is contained in exact one tour), Arkin et al. [1] presented a 6-approximation algorithm.

It is open to develop a constant factor approximation for DVRP or prove there cannot be such an algorithm under some complexity assumption.

References

[1] E.M. Arkin, R. Hassin, and A. Levin, Approximations for minimum and min-max vehicle routing problems, Journal of Algorithms 59 (2006), 1–18.

[2] H. Nagamochi, K. Mochizuki, T. Ibaraki, Complexity of the single vehicle scheduling problems on graphs. *Information Systems and Operations Research* 35 (1997) 256–276.

[3] V. Nagarajan and R. Ravi, Minimum vehicle routing with a common deadline, Diaz, Jansen, Rolim and Zwick (Editors). Workshop on approximation algorithms for combinatorial optimization problems, Springer, 2006, pp. 212–223.