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# Transition Diagram Systems and Normal Form Algorithms*

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**Abstract**

We investigate the complexity of a variety of normal-form transformations for transition diagram systems, which are a parsing view of extended context-free grammars. A transition diagram system is a finite collection of finite-state machines each of which is labeled with a unique nonterminal symbol. The study is motivated by the implementation project GraMa which will provide a C++ toolkit for the symbolic manipulation of context-free objects just as Grail does for regular objects.

## 1 Introduction

In the 1960's, extended context-free grammars (as Backus–Naur form) were introduced as a useful abbreviatory notation that made context-free grammars easier to write. More recently, the Standardized General Markup Language (SGML) [17] used a similar abbreviatory notation to define extended context-free grammars for documents. Currently, XML [4], which is a simplified version of SGML, is being promoted as the markup language for the web, instead of HTML (a specific grammar or DTD specified using SGML). The original development of extended context-free grammars was mirrored by the design of parsers based on them that used what Conway [5] calls transition diagram systems. These systems are simply finite collections of finite-state machines for the right-hand side languages of extended context-free grammars. The difference is, however, that they naturally suggest a parsing approach based on recursive finite-state machines. This idea was used by Glennie [10], by Conway [5], by Kaminger [18], by Lomet [19, 20], as well as by Tixier [26]. More recently, the designers of the document editor Rita [6, 8, 7] used this model to more rapidly prototype

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the editor's syntax analyzer to specific document grammars. They also used it to direct the user's interaction with their structured documents by dynamically generating menus based on the current structure of a document and by generating possible insertion structures in the current context.

We are currently designing a manipulation toolkit GraMa for extended context-free grammars, pushdown machines, and context-free expressions. It is an extension of Grail [22, 21], a similar toolkit for regular expressions and finite-state machines. As a result, we need to design representations of grammars and machines that admit efficient normal-form transformation algorithms (as well as other algorithms of interest). Specifically, as GraMa will use object-oriented inheritance, we have to provide methods for normal-form transformations that will be efficient whatever representation is used for the regular right-hand-side languages in extended context-free grammars. Understanding the complexity of these transformations for transition diagram systems will help us to design an efficient GraMa toolkit. Surprisingly, there does not appear to have been any consolidated effort to study transition diagram systems in their own right apart from the formalization work of Lomet [20]. In a companion paper we investigate the use of regular expressions for the right-hand-side languages [2].

Earlier results on context-free grammars were obtained by Harrison and Yehudai [13, 14, 28] and by Hunt *et al.* [16] among others. Harrrison's chapter [13] on normal form transformations provides an excellent survey of early results. Cohen and Gotlieb [3] suggested a specific representation for context-free grammars and demonstrated how it aided the programming of various operations on them.

We first define extended context-free grammars using the notion of production schemas that are based on finite-state machines.

We then describe algorithms for the fundamental normal-form transformations in Section 3. We then tackle the algorithmic problems for various normal forms of transition diagram systems.

# 2 Notation and terminology

We treat extended context-free grammars as context-free grammars in which the right-hand sides of productions are finite-state machines. Let $V$ be an alphabet and let *lambda* denote the null string. Then, we define a finite-state machine over $V$ and its language in the usual way [1, 27]. We consider machines that are in general nondeterministic and have null-input moves. Moreover, we assume that there is a unique start state and a unique final state and that those states are only-exiting and only-entering, respectively (without loss of generality since we can always add a new start and a new final state to the given machine). We specify a finite-state machine with a tuple $M = (Q, V, E, s, f)$, where $Q$ is the set of states, $s, f \in Q$ are the start and the final states, respectively, and $E \subseteq Q \times V \cup \{\lambda\} \times Q$ is the transition relation. More precisely we will refer to the machine as the corresponding state-transition digraph, where $Q$ is the set of vertices and $E$ is the set of directed labeled edges. Moreover this digraph has a source and a sink vertex that corresponds to the start and final states, respectively. We will often need to determine whether, in a given machine, there is a path from a state $p$ to a state $q$ that corresponds to the label $w \in V^*$. We will denote such a

path by $[p, w, q]$. Furthermore, we will often refer to an edge $(p, X, q)$ as an $X$-edge or as an $X$-transition.

An **extended context-free grammar** $G$ is specified by a tuple $(N, \Sigma, P, S)$, where $N$ and $\Sigma$ are disjoint finite alphabets of **nonterminal symbols** and **terminal symbols,** respectively, $P$ is a finite set of **production schemas,** and the nonterminal $S$ is the **sentence symbol.** Each production schema has the form $A \rightarrow M_A$, where $A$ is a nonterminal and $M_A = (Q_A, V, E_A, s_A, f_A)$ is a finite-state machine over $V = N \cup \Sigma$, for some nonterminal $A$. The **language L(G) of an extended context-free grammar G** is the set of terminal strings derivable from the sentence symbol of $G$. Formally, $L(G) = \{w \in \Sigma^* \mid S \Rightarrow^+ w\}$, where $\Rightarrow^+$ denotes the transitive closure of the derivability relation.

Even though a production schema may correspond to an infinite number of ordinary context-free productions, it is known that extended and standard context-free grammars describe exactly the same languages; for example, see the texts of Salomaa [25] and of Wood [27].

We denote the size of a set $A$ by $\#A$. We define the size $|M|$ of a finite-state machine $M = (Q, V, E, s, f)$ by $|M| = |E| = \sum_{(p,X,q) \in E} |pXq|$, that is, as the size of the corresponding state digraph[1].

To measure the complexity of any grammatical transformation we need to define the size of a grammar. There are two traditional measures of the size of a context-free grammar that we extend to extended context-free grammars as follows. Given an extended context-free grammar $G = (N, \Sigma, P, S)$, we define the **size of G** to be

$$\sum_{A \in N} (1 + |M_A|)$$

and we denote it by $|G|$. We define the **norm of G** to be

$$|G| \log \#(N \cup \Sigma)$$

and we denote it by $\| G \|$. Clearly, the norm is a more realistic measure of a grammar's size as it takes into account the size of the encoding of the symbols of the grammar but we use only the size measure here.

# 3 Normal-form transformations

As an extended context-free grammar has a number of production schemas that are finite-state machines, we represent it as a labeled digraph and the set of all start–final pairs $(s_A, f_A)$ of all the machines. Given a nonterminal $A$ and its start–final pair $(s_A, f_A)$, the subgraph that includes all connected paths from $s_A$ to $f_A$ gives the production schema for $A$. The representation has been chosen to avoid subgraph repetition.

## 3.1 Reachability and usefulness

A symbol $X$ is **reachable** if it appears in some string derived from the sentence symbol; that is, if there is a derivation $S \Rightarrow^+ \alpha X \beta$ where $\alpha$ and $\beta$ are possibly null strings over $\Sigma \cup N$.

[1] More precisely, $|M| = 3\#E$.

To solve the reachability problem for grammars, we first ensure that, for each nonterminal $A$, its machine $M_A$, has only reachable and useful states. As in standard context-free grammars, reachable symbols can be easily identified by means of a digraph traversal. More precisely, we construct a digraph whose vertices are symbols in $N \cup \Sigma$ and there is an edge from $A$ to $B$ if and only if $B$ labels an edge in the machine $M_A$. Then, a depth-first traversal of this digraph starting from $S$ gives all reachable symbols of the grammar. The time taken by the traversal and digraph construction is linear in the size of the grammar. (In practice, we do not explicitly construct the reachability digraph.)

A nonterminal symbol $A$ is **useful** if there is a derivation $A \Rightarrow^+ \alpha$, where $\alpha$ is a terminal string. The set of useful symbols can be computed iteratively as follows. First, all terminal symbols are useful. Second, in one pass through the grammar, we find all nonterminals $B$ such that in $M_B$ there is some path $[s_B, w, f_B]$, where $w$ is a string (possibly null) of terminal symbols. All such nonterminals $B$ are useful symbols. Next, an unclassified nonterminal $A$ is useful if, in $M_A$, there is a path $[s_A, w, f_A]$, where $w$ is a string of currently useful symbols. We now carry out another pass through the grammar until no unclassified nonterminals are demonstrated to be useful. The algorithm, as described, runs in $O(\#N|G|)$ time.

We can speed up the preceding algorithm such that it runs in time linear in $|G|$ as follows. In the first pass, we first use what we call a **blocking traversal** of each finite-state machine $M_A$ from its start state $s_A$. A blocking traversal is a standard breadth-first traversal except that whenever it reaches edges that contain unclassified nonterminals, it does not follow them; it treats these edges as if they are not in the digraph. If $(p, B, q)$ is such a blocking edge, then the traversal also adds $q$ to the continuing set $\text{cont}(B)$ for $B$. (Before beginning the traversals, we initialize all continuing sets to be empty.) As a result, the only edges that the initial blocking traversal explores are ones that are labeled with either terminals or the null string. Therefore, if the traversal ever visits the final state $f_A$, then we have demonstrated that $A$ is useful.

After we finish the initialization pass, some nonterminals are classified as useful. (If none are, then $L(G)$ is empty.) If there are only empty continuing sets, then we have found all useful nonterminals and the algorithm terminates. Otherwise, we carry out the second pass. For all useful nonterminals $B$, we continue the blocking traversals, in all the machines, from the states in the continuing set of $B$ since the edges that were previously blocked can now be followed. We also remove these states from their continuing sets. As before, whenever a blocking traversal reaches edges that contain unclassified nonterminals, it does not follow them; it treats them as if they are not in the digraph. If $(p, C, q)$ is such a blocking edge, then the traversal also adds $q$ to the continuing set $\text{cont}(C)$. As a result, the only edges that the blocking traversal explores are ones that are labeled with either useful symbols or the null string. We again continue these partial traversals until none of them can continue. Any one of the partial traversals may meet a final state and demonstrate that the corresponding nonterminal is useful. We repeat passes until no more unclassified nonterminals are demonstrated to be useful. As we keep the partial traversals at all times, we can ensure that we never explore an edge more than once. Thus, the algorithm runs in $O(|G|)$ time.

A grammar $G$ is **reduced** if all its symbols are both reachable and useful. As in standard context-free grammars, to reduce a grammar we first identify all useful symbols and then

select (together with the corresponding schemas) those symbols (and their schemas) that are also reachable.

## 3.2 Null-free form

Given an extended context-free grammar $G = (N, \Sigma, P, S)$, we can determine the nullable nonterminals (the ones that derive the null string) using a similar algorithm to the one we used for usefulness, see Section 3.1. This algorithm takes $O(|G|)$ time. Given this information we can make the grammar null free in two steps. First, for each nullable symbol $A$ and for all edges $(p, A, q)$ (in all the machines), we add new edges $(p, \lambda, q)$. This step takes time proportional to the total numbers of appearances of nullable symbols in $G$ and at most doubles the size of the grammar.

In the second step we transform each production schema $A \rightarrow M_A$, where $A$ is nullable, into a null-free production schema $A \rightarrow M'_A$, where $\lambda \notin L(M'_A)$. This step takes time $O(|G|)$ in the worst case and produces a new grammar that has size $O(|G|)$.

We now describe this transformation. Let $M = (Q, V, E, s, f)$ be a machine with $\lambda \in L(M)$ that we wish to transform such that its language does not contain the null string. The idea is to guarantee that each path from $s$ to $f$ contains at least one non-null symbol. One way is to ensure that all paths from $s$ to $f$ start with a non-null labeled edge. Notice that, since $s \neq f$, $\lambda \in L(M)$ implies that there is at least one null path from $s$ to $f$. We consider the subgraph of $M$ that corresponds to all null paths from $s$ and refer to its set of vertices as $S^\lambda$. Then, we ensure that all the paths in $M$ start with a non-null labeled edge. More formally, we define the following transformation:

**Null-removal from the start:** *Let $S^\lambda$ be the set of states reachable from $s$ by null paths. Delete all edges $(s, \lambda, p)$ and, for each edge $(p, B, q) \in E$ with $B \neq \lambda$, $p \in S^\lambda$ and $p \neq s$, add a new edge $(s, B, q)$ to $E$.*

After applying this transformation, we say the machine is in **start-null-free form.**

Alternatively, we can achieve the same result by ensuring that all paths from $s$ to $f$ end with a non-null labeled edge. We define this complementary transformation as follows.

**Null removal to the final:** *Let $F^\lambda$ be the set of states from which $f$ is reachable by null paths. Delete all edges $(q, \lambda, f)$ and, for each edge $(p, B, q) \in E$ with $B \neq \lambda$, $q \in F^\lambda$ and $q \neq f$, add a new edge $(p, B, f)$ to $E$.*

After applying this transformation, we say the machine is in **final-null-free form.** It is straightforward to verify that applying one of these two transformations to a given machine $M$ whose language contains the null string, will remove the null string from the language. Moreover the transformation takes time $O(|M|)$, in the worst case, and doubles, at most, the size of $M$.

Thus, performing the second step for null-productions removal, will take a total time of $O(|G|)$ with only a linear increase in the size of $G$.

## 3.3 Unit-free form

A **unit production** is a production of the form $A \rightarrow B$. We assume that the given extended context-free grammar is null-free, in which case, to transform it into unit-free form requires

three steps. First, we identify all unit productions. Second, we transform the finite-state machines such that their languages do not include unit-nonterminal strings. Lastly, we modify the production schemas such that whenever $A \Rightarrow^+ B_i$, $1 \leq i \leq h$, in the original grammar, for some $h \geq 0$, ($B_i$ is unit reachable from $A$), we replace $A \rightarrow M_A$ in the original grammar with $A \rightarrow M'_A$, where $L(M'_A) = L(\bar{M}_A) \cup \bigcup_{i=1}^{h} L(M_{B_i})$ and $L(\bar{M}_A) = L(M_A) \setminus \{B\}$.

Given an extended context-free grammar $G$, for each nonterminal symbol $A$, $A$'s unit productions correspond to nonterminal strings of length one accepted by the corresponding machine $M_A$.

As a first step, we transform each machine $M = (Q, V, E, s, f)$ into both *start-null-free form* and *final-null-free form.* This takes time $O(|M|)$, in the worst case. It simplifies both the detection and the removal of unit-nonterminal strings accepted by $M$. In particular such strings correspond to edges $(s, B, f)$, where $B$ is a nonterminal symbol. We refer to $(s, B, f)$ as a unit edge. Next, for all nonterminals $A$, we determine the set $U_A$ of all nonterminals that are unit reachable from $A$. These sets are needed to modify the production schemas. The running time for this step is $O(|G|)$.

Now, in the second step, we remove all edges of the form $(s_A, B, f_A)$ from machine $M_A$, where $B$ is a nonterminal symbol. Since $s_A$ is only exiting and $f_A$ is only entering, this removal only removes $B$ from $L(M_A)$; it does not remove any other strings from $L(M_A)$. Finally, let $M''_A = (Q_A, V, E''_A, s_A, f_A)$ be the unit-free machines of the original grammar $G$. The time to perform the second step is $O(|G|)$.

We modify the production schemas such that, for each nonterminal $A$, if $B_1, \ldots, B_h$ are all symbols unit reachable from $A$, then the new production schema is $A \rightarrow M'_A$, where

$$L(M'_A) = L(M''_A) \cup L(M''_{B_1}) \cup \cdots \cup L(M''_{B_h}).$$

To avoid duplicating the state digraphs which would give a quadratic blow up in the size of the resulting grammar in terms of $|G|$, we define the new machines $M'_A = (Q'_A, V, E'_A, s'_A, f'_A)$ as follows:

$$\begin{aligned} Q'_A &= Q_A \cup Q_{B_1} \cup \cdots \cup Q_{B_h} \cup \{s'_A, f'_A\}, \\ E'_A &= E''_A \cup E''_{B_1} \cup \ldots \cup E''_{B_h} \\ &\quad \cup \{(s'_A, \lambda, s_{B_i}) : 1 \leq i \leq h\} \\ &\quad \cup \{(f_{B_i}, \lambda, f'_A) : 1 \leq i \leq h\} \\ &\quad \cup \{(s'_A, \lambda, s_A), (f_A, \lambda, f'_A)\}. \end{aligned}$$

We have added a new start state and a new final state to $M''_A$. We have avoided making duplicate copies of the machines $M_{B_i}$ by adding the null edges from the new start state to the start states of all original machines that are involved. This modification ensures that we can enter any of these machines when determining a specific production for $A$. More important, by adding null edges from the final states of all involved original machines to the new final state, we guarantee that only strings that determine a path from the new start state to the new final state yield a production for $A$. There are at most $2\#N$ new null edges for each machine $M'_A$; therefore, this step takes time $O(\#N^2)$ with an increase of $O(\#N^2)$ in the size of the grammar $G$.

In total, the proposed null-removal algorithm takes $O(|G|)$ time and space.

## 3.4 Greibach form

The original normal form result for context-free grammars was established by Sheila Greibach [11] in the 1960's; it was a key result in the use of the multiple-path syntactic analyzer developed at Harvard University at that time. An extended context-free grammar is in *Greibach normal form* if its productions have only the following form:

$$A \rightarrow a\alpha,$$

where $a$ is a terminal symbol and $\alpha$ is a possibly empty string of nonterminal symbols. The transformation of an extended context-free grammar into Greibach normal form requires two steps: left-recursion removal and back left substitution. Recall that a grammar is left recursive if there is a nonterminal $A$ such that $A \Rightarrow^+ A\alpha$ in the grammar, for some string $\alpha$. We consider the second step first.

We assume that the given extended context-free grammar $G = (N, \Sigma, P, S)$ is reduced, factored, null free, and unit free. In addition, for the second step, we also assume that the grammar is not left recursive. Since the grammar is not left recursive there is a partial order on the nonterminals, **left reachability,** that is defined by $A \leq B$ if there is a leftmost derivation $A \Rightarrow^* B\alpha$. As usual, we can consider the nonterminals to be enumerated as $A_1, \ldots, A_n$ such that whenever $A_i \leq A_j$, then $i \leq j$. Observe that $A_n$ is already in Greibach normal form since it has only productions of the form $A_n \rightarrow a$, where $a \in \Sigma$. We now convert the nonterminals one at a time from $A_{n-1}$ down to $A_1$. The conversion is conceptually simple, yet it could be computational expensive. When converting $A_i$, we replace all nonterminals that can appear in the first positions in the strings in $L(M_{A_i})$ with a copy of their machines. (If we do this replacement explicitly, we get a quadratic blow-up in the size of $G$.) Thus, $A_i \rightarrow M'_A$ is now in Greibach normal form.

To carry out the replacement efficiently we first convert each finite machine $M_i$ into start-null-free form, which takes linear time and space with respect to $|G|$ (see Section 3.2). Then, to avoid quadratic blow up we can do an implicit substitution of the nonterminal symbol by its machine as follows. For each machine $M = (Q, V, E, s, f)$, we clone both its start and the final states. More precisely, we rename $s$ and $f$ as $\bar{s}$ and $\bar{f}$, respectively, and refer to them as dummy start and dummy final states, respectively. Then, we connect the dummy states to their corresponding real states by defining new null edges $(s, \lambda, \bar{s})$ and $(\bar{f}, \lambda, f)$. Now to substitute (implicitly) an edge $(\bar{s_A}, B, q)$ with the corresponding machine $M_B$ we add only edges $(\bar{s_A}, \lambda, \bar{s_B})$ and $(\bar{f_B}, \lambda, q)$, and remove the edge $(\bar{s_A}, B, q)$. It gives only a linear increase in the size of the grammar.

If the grammar is left recursive, we first need to remove left recursion. We use a technique introduced by Greibach [12], investigated in detail by Hotz and his his co-workers [15, 23, 24] and rediscovered by others [9]. It involves producing, for each nonterminal, a distinct subgrammar of $G$ that is left linear and, hence, can be converted into an equivalent right linear grammar. (Although this specific approach was not used by Greibach, she used a similar approach.) This conversion changes left recursion into right recursion and does not introduce any new left recursion. For more details, see Wood's text [27]. The important property of the left-linear subgrammars is that every sentential leftmost derivation sequence in $G$ can be mimicked by a sequence of leftmost derivation sequences, each of which is

a sentential leftmost derivation sequence in one of the left-linear subgrammars. Once we convert the left-linear grammars into right-linear grammars this property is weakened in that we mimic the original derivation sequence with a sequence of sentential rightmost derivation sequences in the right-linear grammars. The new grammar that is equivalent to $G$ is the collection of the distinct right-linear grammars, one for each nonterminal in $G$.

As the modified grammar is no longer left recursive, we can now apply back left substitution to obtain a final grammar in Greibach normal form.

We now analyze the complexity of the described algorithm. Left-recursion removal causes a blow up of $\#N|G|$ at worst in the size of the grammar and squares the number of nonterminals from $\#N$ to $\#N^2$. Then, to avoid a blow up of $\#N|G|$ in the size of the grammar by doing back left substitution explicitly, we use the preceding implicit technique. In this case, the only expensive (that is, nonlinear) step is the left-recursion removal; thus, the complete conversion time and space is reduced to $O(\#N|G|)$.

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