

Iterative Combinatorial Auctions: Theory and Practice

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AAAI-00, pp. 74-81

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May 2006
COMP6700
HKUST

Why iterative?

- In Generalized Vickrey Auction (GVA)
 - $2^{|G|}$ bundles to evaluate
 - A large winner-determination (WD) problem
 - Optimal
- In iBundle
 - Agents bid for less bundles
 - Each iteration contains a smaller WD problem
 - Also optimal for reasonable agent bidding strategy

Outline

- iBundle – the model
- Optimality
- Computational Analysis

Notation

G denote the set of items to be auctioned,

I denote the set of agents,

$S \subseteq G$ denote a bundle of items.

Bid – each bid contains a *buddle* S and its corresponding *bid price*

For each buddle S in round t , there are

$p_{\text{bid},i}^t(S)$ - *bid price*, provided by agent i

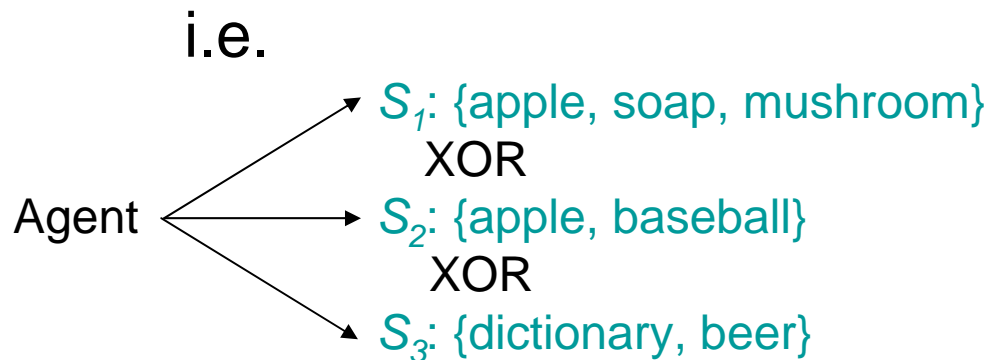
$p_{\text{ask}}^t(S)$ - *ask price*, announced by the auctioneer

Bidding Price

- $p_{bid,i}^t(S) \geq p_{ask}^t(S) - \varepsilon$, where $\varepsilon > 0$ is the minimal increment of ask price.
- Agent can also bid ε below the ask price, but then it cannot bid a higher price in future rounds.
- Agent will NOT bid a price that higher than the value of the bundle, i.e. $p_{bid,i}^t(S) \leq v_i(S)$

XOR Bids

- Agents can place **multiple** bids for **exclusive-or** bundles, e.g. S_1 XOR S_2 , to indicate that an agent wants either all items in S_1 or all items in S_2 but **not both** S_1 and S_2 (The allocation guarantees a agent is satisfied by at most one bid (bundle) or nothing).



The Auction - iBundle

Winner-determination. The auctioneer solves a winner-determination problem in each round, computing an allocation of bundles to agents that maximizes revenue. The auctioneer must respect agents' XOR bid constraints, and cannot allocate any item to more than one agent. The provisional allocation becomes the final allocation when the auction terminates.

Termination. The auction terminates when: [T1] all agents submit the same bids in two consecutive rounds, or [T2] all agents that bid receive a bundle.

Approximate WD

Approximate Winner-determination. The auctioneer can also use an approximate algorithm for winner-determination, and still maintain the same incentives for myopic agents to follow the same bidding strategy. To achieve this an approximate algorithm must have the *bid-monotonicity* property:

Monotonicity¹ $s \subseteq g_j, \quad s' \subseteq s, \quad v' \geq v \Rightarrow s' \subseteq g'_j.$

People will not get lose his bid if he propose more money for fewer goods.

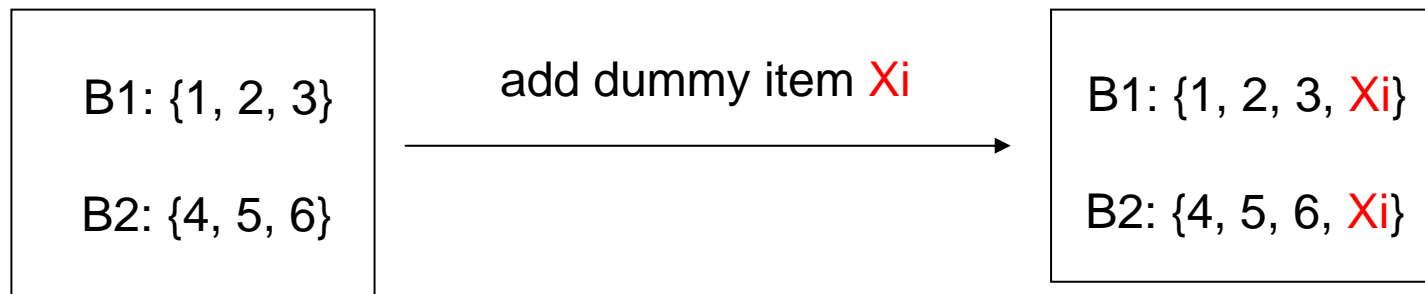
¹ From Lehmann *et al.*(1999)

Myopic Best-Response Bidding Strategy

- A myopic agent bids to maximize utility at the current ask prices.
- The myopic best-response strategy is to submit an XOR bid for all bundles S that maximize utility $u_i(S) = v_i(S) - p(S)$ at the current prices. This maximizes the probability of a successful bid for bid-monotonic WD algorithms.

Dummy item

- When agent i have **un-conflict** bids, add an **dummy item** to each bids, so that all those bids will not be compatible with each other (so that the XOR bids don't need special care)



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Optimality of iBundle

Theorem 1. *iBundle terminates with an allocation that is within $3 \min\{|G|, |I|\}\epsilon$ of the optimal solution, for myopic best-response agent bidding strategies.*

Proof of Theorem 1

- IP Model

$$\max_{x_i(S)} \sum_{S \subseteq G} \sum_{i \in I} x_i(S) v_i(S) \quad [\text{IP}]$$

$$\text{s.t.} \quad \sum_{S \subseteq G} x_i(S) \leq 1, \quad \forall i \quad (\text{IP-1})$$

$$\sum_{S \subseteq G, j \in S} \sum_i x_i(S) \leq 1, \quad \forall j \quad (\text{IP-2})$$

$$x_i(S) \in \{0, 1\}, \quad \forall i, S$$

$x_i(S)$ indicate whether or not agent i receives bundle S

$v_i(S)$ denote agent i 's value for bundle S

Proof of Theorem 1

- LP Relaxation

$$\max_{x_i(S), y(k)} \sum_{S \subseteq G} \sum_{i \in I} x_i(S) v_i(S) \quad [\text{LP}_2]$$

$$\text{s.t.} \quad \sum_{S \subseteq G} x_i(S) \leq 1, \quad \forall i \quad (\text{LP-1})$$

$$\sum_{i \in I} x_i(S) \leq \sum_{k \in K, S \in k} y(k), \quad \forall S \quad (\text{LP-2})$$

$$\sum_{k \in K} y(k) \leq 1 \quad (\text{LP-3})$$

$$x_i(S), y(k) \geq 0, \quad \forall i, S, k$$

k correspond to a partition of items into bundles

K is the set of all possible partitions

Proof of Theorem 1

- LP Dual

$$\min_{p(i), p(S), \pi} \sum_{i \in I} p(i) + \pi \quad [\text{DLP}_2]$$

$$\text{s.t. } p(i) + p(S) \geq v_i(S), \quad \forall i, S \quad (\text{DLP-1})$$

$$\pi - \sum_{S \in k} p(S) \geq 0, \quad \forall k \quad (\text{DLP-2})$$

$$p(i), p(S), \pi \geq 0, \quad \forall i, S$$

		$\max_{x_i(S), y(k)} \sum_{S \subseteq G} \sum_{i \in I} x_i(S) v_i(S)$	$[\text{LP}_2]$
$p(i)$	\longrightarrow	$\sum_{S \subseteq G} x_i(S) \leq 1, \quad \forall i$	(LP-1)
$p(S)$	\longrightarrow	$\sum_{i \in I} x_i(S) \leq \sum_{k \in K, S \in k} y(k), \quad \forall S$	(LP-2)
π	\longrightarrow	$\sum_{k \in K} y(k) \leq 1$	(LP-3)
		$x_i(S), y(k) \geq 0, \quad \forall i, S, k$	

Proof of Theorem 1

Feasible primal. To construct a feasible primal solution assign $x_i(S_i) = 1$ and $x_i(S') = 0$ for all $S' \neq S_i$. Partition $y(k^*) = 1$ for $k^* = [S_1, \dots, S_{|I|}]$, and $y(k) = 0$ otherwise.

Feasible dual. To construct a feasible dual solution assign $p(S) = p_{\text{ask}}(S)$. Constraints (DLP-1) and (DLP-2) are satisfied with assignments:

$$p(i) = \max \left\{ 0, \max_{S \subseteq G} \{v_i(S) - p(S)\} \right\} \quad (1)$$

$$\pi = \max_{k \in K} \sum_{S \in k} p(S) \quad (2)$$

$p(i)$ can be interpreted as agent i 's maximum utility

π can be interpreted as the maximum revenue

Complementary-slackness conditions

- $x_i(S) > 0 \Rightarrow p(i) + p(S) = v_i(S), \quad \forall i, S$

$$v_i(S) - p_{\text{ask}}(S) + 2\epsilon \geq \max \left\{ 0, \max_{S'} \{v_i(S') - p_{\text{ask}}(S')\} \right\}$$

$$p(i) + p(S) \leq v_i(S) + 2\epsilon, \quad \forall i, S$$
- $y(k) > 0 \Rightarrow \pi - \sum_{S \in k} p(S) = 0, \quad \forall k$

$$\pi - \sum_{S \in k} p(S) \leq \min\{|G|, |I|\}\epsilon, \quad \forall k$$

	$\min_{p(i), p(S), \pi} \sum_{i \in I} p(i) + \pi$	[DLP ₂]
$x_i(S)$	$\longrightarrow p(i) + p(S) \geq v_i(S), \quad \forall i, S$	(DLP-1)
$y(k)$	$\longrightarrow \pi - \sum_{S \in k} p(S) \geq 0, \quad \forall k$	(DLP-2)
	$p(i), p(S), \pi \geq 0, \quad \forall i, S$	

Optimality

$$\sum_{i \in I} p(i) \leq \sum_{i \in I} v_i(S_i) - \sum_{i \in I} p(S_i) + 2 \min\{|G|, |I|\} \epsilon$$

$$\pi \leq \sum_{i \in I} p(S_i) + \min\{|G|, |I|\} \epsilon$$

$$\pi + \sum_{i \in I} p(i) \leq \sum_{i \in I} v_i(S_i) + 3 \min\{|G|, |I|\} \epsilon$$

$$V_{\text{DL P}} \leq V_{\text{LP}} + 3 \min\{|G|, |I|\} \epsilon \quad V_{\text{LP}}^* \leq V_{\text{DL P}}$$

$$V_{\text{LP}} \geq V_{\text{LP}}^* - 3 \min\{|G|, |I|\} \epsilon$$

The constructed primal solution is integral, so V_{LP} is feasible and optimal

Termination

Termination. By contradiction, assume the auction never terminates. Informally, [T1] implies that agents must submit different bids in successive rounds, but with myopic best-response bidding this implies that prices must increase, and agents must eventually bid above their values for bundles. We prove a contradiction with myopic best-response bidding strategies.

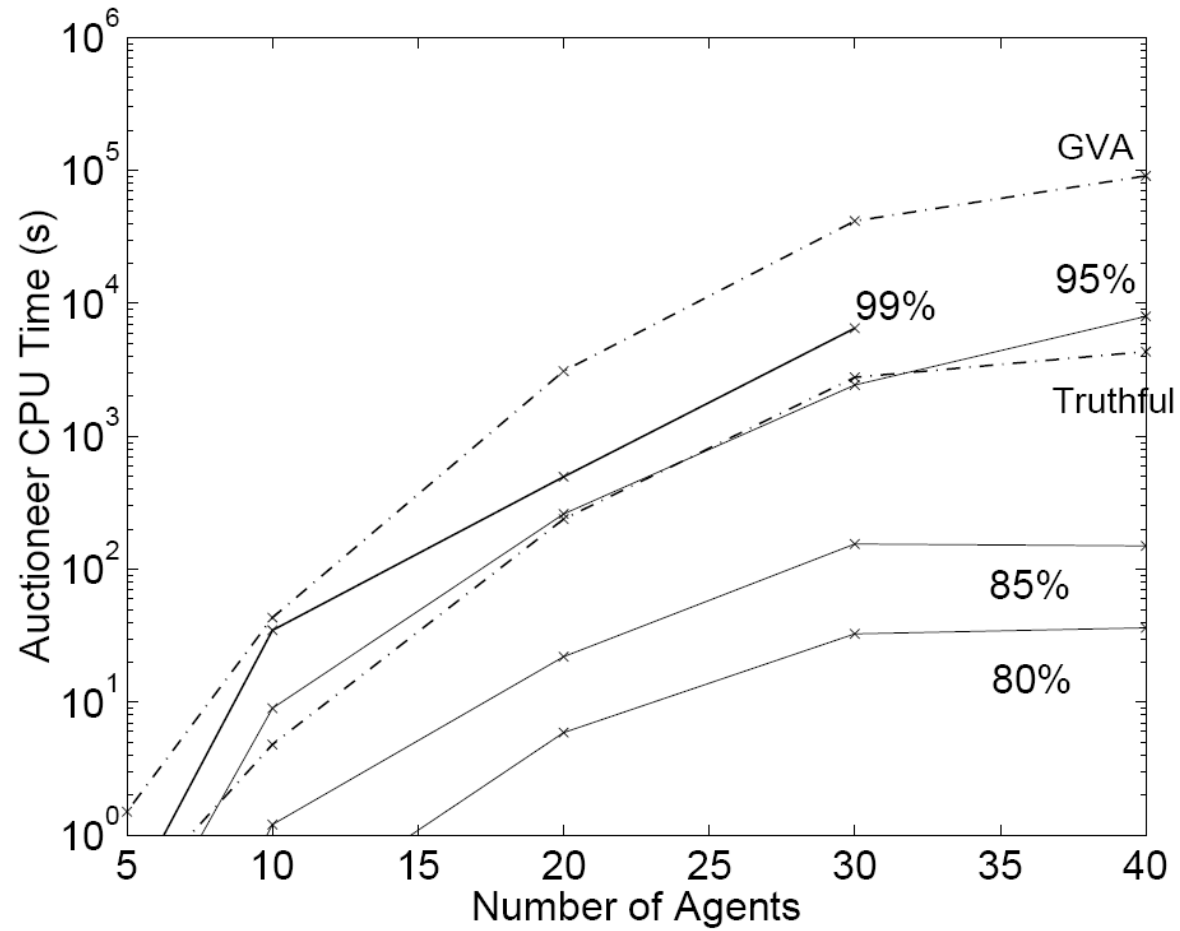
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Computational Analysis

- In iBundle, the worst case gives $O(BV_{max}/\varepsilon)$ to converge
- The value of ε determines
 - The number of rounds to termination
 - The allocative efficiency

Computational Results



Computational Results

Problem		GVA	<i>i</i> Bundle			Approx- Bundle
			$\simeq 90\%$	$\simeq 95\%$	$\simeq 99\%$	
Decay 67.3% ^d 13.4 ^e	Eff (%)	100	91.5	94.9	98.3	85.1
	WD-time ^a (s)	41700	831	2400	5650	0
	Pr-time ^b (s)	–	26	34.5	44	39.2
	Comm ^c (kBit)	18.8	221	306	394	377
WR 71.5% 1	Eff (%)	100	90.7	94.9	99.2	79.4
	WD-time (s)	3	0.6	1.7	6	0
	Pr-time (s)	–	5.4	11.5	40.9	12.2
	Comm (kBit)	18.1	20.5	52.1	144	53.1
Rand 37.8% 11.2	Eff (%)	100	89.3	97	99	95.8
	WD-time (s)	68	4.4	7.4	11	0
	Pr-time (s)	–	6.5	9.7	12.1	12.9
	Comm (kBit)	18.7	49.5	66.4	82.6	85.6
Unif 58% 3	Eff (%)	100	–	95.6	99.1	76.2
	WD-time (s)	25	–	6.6	18.7	0
	Pr-time (s)	–	–	14.7	42.0	46
	Comm (kBit)	18.2	–	56.5	120	124