1. **Open Addressing**
   Let table size be $m = 15$ (with items indexed from $0 \ldots 14$).
   Use the hash function $h(x) = (x \mod 15)$ and linear hashing to hash the items 19, 6, 18, 34, 25, 34 in that order.
   Draw the resulting table.

2. **Universal Hashing**
   Recall the universal hash function family defined by
   
   $$h_{a,b}(x) = \left( (ax + b) \mod p \right) \mod m$$

   where $a \in \mathbb{Z}^*_p$, $b \in \mathbb{Z}_p$ and $p$ is a prime with $p \geq U$. Let $p = 17$, $m = 5$. For all $x = 0, 1, \ldots, 16$ write the values for $h_{1,0}(x)$. Now write all the values for $h_{2,2}(x).$
3. Divide and Conquer for closest pair

Let $P = \{p_1, p_2, \ldots, p_n\}$ be $n$ two-dimensional points and define

$$\delta(P) = \min_{p, p' \in P: p \neq p'} d(p, p')$$

to be the closest pair distance of $P$.

Let $X$ be a real value and split $P$ on the line $x = X$ so that

$$P_L = \{p \in P : p.x \leq X\}, \quad P_R = \{p \in P : p.x > X\}.$$

Suppose you are given the closest pair distance of the two sets:

$$\delta_L = \delta(P_L) \quad \text{and} \quad \delta_R = \delta(P_R).$$

Set $\delta' = \min(\delta_L, \delta_R)$ and define the points contained by the $\delta'$ strips to the left and right of the line $x = X$ by

$$S_L = \{p \in P_L : X - p.x \leq \delta'\}, \quad S_R = \{p \in P_R : p.x - X \leq \delta'\}$$

(a) Prove that

$$\delta(P) = \min(\delta_L, \delta_R, d(S_L, S_R))$$

where $d(P_1, P_2) = \min\{d(p_i, p_k), : p_1 \in P_1, p_2 \in P_2\}$.

(b) Suppose that you are given the values $\delta_L$ and $\delta_R$ and each of the sets $P_L$ and $P_R$ sorted by $y$-coordinate. Show how to calculate $\delta(P) = \min(\delta_L, \delta_R, d(S_L, S_R))$ in $O(n)$ time.

Hint. In $O(n)$ time first find $S_L$ and $P_L$, each sorted by $y$ coordinate. Then show how, in $O(|S_L| + |S_R|)$ time, you can find $d(S_L, S_R)$ by using the ideas from the gridding lemma.

(c) Now construct a divide and conquer algorithm for finding $\delta(P)$ that works by

(i) Finding the median by $x$-coordinate of $P$. Set this $x$ coordinate to be $X$.

(ii) Split $P$ on $X$ into $P_L$ and $P_R$.

(iii) Recursively find $\delta(P_L)$ and $\delta(P_R)$

(iv) Use the ideas above to find $\delta(P)$ using $O(n)$ extra time

Note that the recursion will terminate when $P = \{p\}$ or $P = \{p, p'\}$. In those cases $\delta(P) = \infty$ or $\delta(P) = d(p, p')$ can be found in $O(1)$ time.

The correctness of the algorithm follows from (a) and (b).

Show how to implement the algorithm in $O(n \log^2 n)$ time.

(d) Can you improve this to $O(n \log n)$ time?