Statistical Arbitrage with Kalman Filter and Cluster-based Stock Selection

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Abstract

This report aims to analyse a statistical arbitrage trading strategy with Kalman filter. Novel machine learning techniques and tools, e.g. PCA and DBSCAN, are implemented to capture profitable pairs among all possible pairs in US equities. The hedge ratio of pairs will be calculated by estimated parameters by Kalman filter regression. Transaction costs and market frictions have been considered thoroughly in order to make the result more meaningful. Empirical results show that has a higher annual Sharpe ratio (0.95 vs. 0.70), three pairs out of the four tested has a higher annual return and that they have a relative higher cumulative return (48.28% vs. 28.40%) with a slightly higher average volatility.

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Introduction

Pair trading has long been a popular statistical arbitrage strategy. A pair is defined as two assets that have a cointegrating and mean reverting relationship. The strategy consists in exploiting the mispricing of assets and open a long-short position of the paired assets when mispricing between two prices paths is observed.

To set up the strategy, there are two parts: the filtering part that eliminates a majority of pairs, and the implementation of the trading strategy. While pair trading can neglect beta of the market and generate uncorrelated returns (alpha) with minimal exposure to the market, the identification of pairs and implementation are complex, and the cointegrating relationship can break in a short period of time.

One of the most important issues is how to find valid and eligible pairs which exhibit unconditional mean-reverting behaviour and the fact that the problem can get computationally difficult: there are about 5000 companies being traded across exchanges in the US, not to mention other exchange-tradable products derived by mimicking index, holding a basket of stocks as well as tracking price movement of commodities. While pairs can be chosen either based on fundamentals or statistics, an approach combining both will be used. The goal is to have an automated procedure with a high accuracy to select for pairs that will generate a positive PnL on average.

In the first section, an efficient way to filter a large number of pairs using some fundamental factors and tools within machine learning will be investigated. In the second section, a trading strategy as well as its trading rules will be implemented. Lastly, the results will be presented and directions for improvement in the pair selecting model will be discussed.

Software Framework and Source of Data

Quantopian, a platform integrated a Python algorithmic trading library, Zipline, developed by Quantopian Inc, provides a close-to-reality system for back-testing. IPython and other popular data science libraries, such as Pandas, NumPy and StatsModels, for building and executing trading strategies in an integrated manner.

They provide up to minute-level data of all US stocks from January 2002 to the previous trading day. The data is cleaned and the close price is adjusted for corporate actions. By using the platform, pairs can be selected based on fundamentals. The price data includes all companies that were traded, including companies that have subsequently stopped trading, which is very important in avoiding survivor bias.

However, some cointegration test functions, e.g. Johansen Test that tests multiple instruments at a time, cannot be realized with Python packages. Augmented Dickey–Fuller (ADF) test from StatsModels is used in this project.

Cluster-based Pair Selection

Pairs Pre-processing

The default Q3000US US equity universe which includes 3000 tradable stocks is used to filter stock retrieved on 31st December 2013. Pipeline will filter out small-cap stocks that usually come with high volatility, conglomerates stocks that are bad members of any pairs with regard to their multiple revenue streams [1], and stocks with a bad financial health grade that have a higher probability to go bankrupt during backtesting or forward testing, with the database of Morningstar Inc.. The stocks without full return series will also be screened; another approach could be filling the missing data of return series by interpolation. The underlying assumption is that "Stocks that share loadings to common factors in the past should be related in the future" [2]. The stock pool has now reduced to 1508 from 2331 while possible pairs reduced to 1.1 millions from 2.7 millions.

Principal component analysis

In order to select the most profitable candidates for pairs-trading, the universe is clustered on each stock-by their daily return with Principal component analysis (PCA), financial health grade and market cap. PCA is a procedure that uses a transformation to convert a set of observations of possibly correlated attributes into a set of linearly uncorrelated attributes called principal components. PCA is efficient and accurate in reducing dimensions of the returns data while keeping the information.

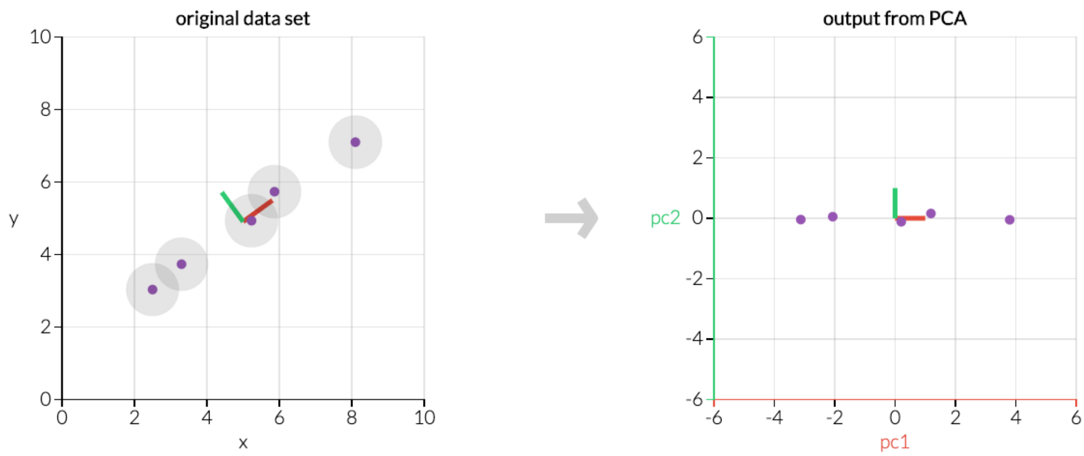

FIGURE 1 – DEMONSTRATION OF PCA

Figure 1 is a demonstration of running a PCA. In the setup, the returns data will be reduced into 40 principal components and will stack up with the fundamental factors, e.g. financial health, market cap. The data pre-processing will be wrapped up by standardising the features.

Clustering, Testing and Stock Selection

Density-based Spatial Clustering of Applications with Noise

Density-based Spatial Clustering of Applications with Noise (DBSCAN) is a density-based clustering non-parametric algorithm: given a set of points, it groups together points that are closely packed together with a predefined radius of a neighbourhood, marking as outliers points that lie outside any neighbourhood. Compared to another popular approach, K-means clustering, the benefit is that the number of clusters needs not to be defined before clustering. Also, DBSCAN does not cluster all stocks, making it appropriate for screening a large number of data points like dataset of thousands of stocks. However, DBSCAN is more computationally intensive than K-means by its higher complexity.

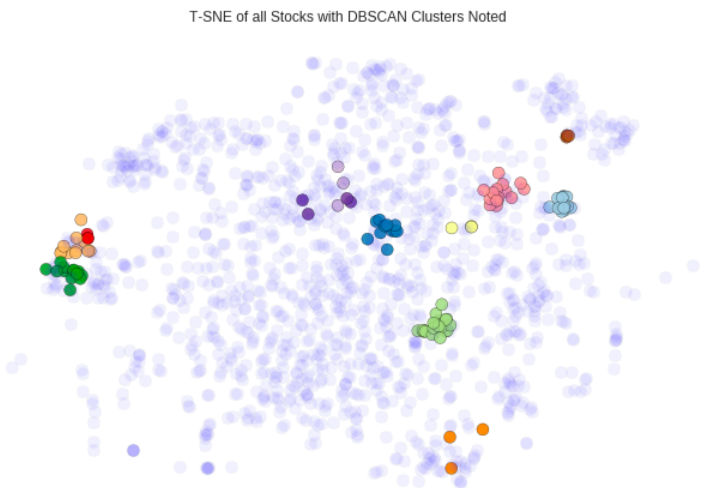

FIGURE 2 – VISUALIZATION OF DBSCAN RESULT

Figure 2 presents the resulting 12 clusters by T-distributed Stochastic Neighbour Embedding (t-SNE) and the optimum parameters used here are "eps = 1.86", "min_samples = 3" after trials. (eps is the maximum distance between two samples for being considered as in the neighbourhood of the other and min_samples is the minimum number in a cluster.)

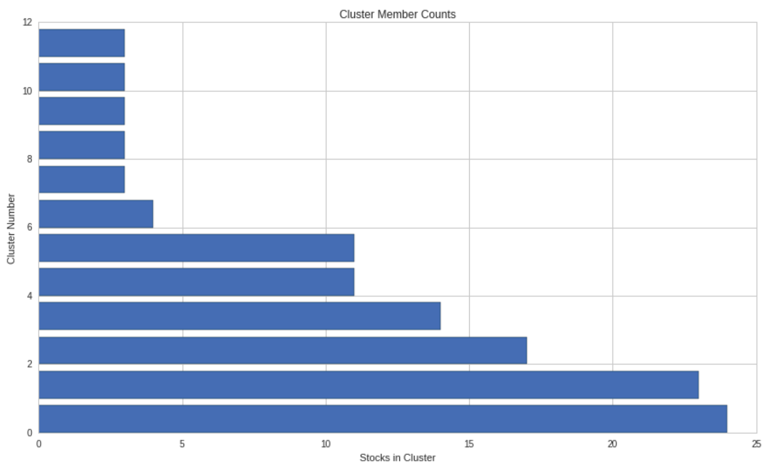

FIGURE 3 – TABLES OF 12 CLUSTERS

Figure 3 presents 10 clusters that have a reasonably small size. Each cluster can then be studied individually.

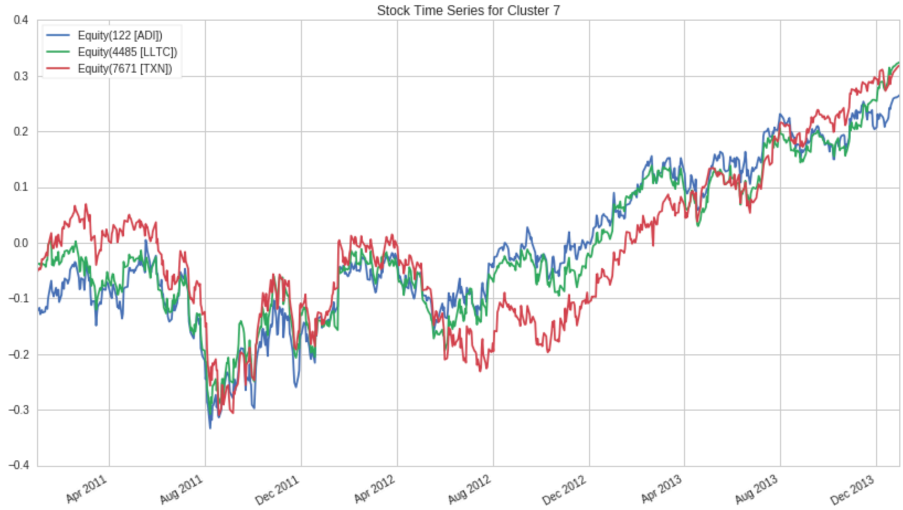

FIGURE 4 – MEMBERS OF CLUSTER 7

In Figure 4, it is not difficult to notice that all three stocks show a strong correlation between each other. However, further cointegration test needed to be carried out to identify the potential pairs.

Augmented Dickey–Fuller Test

Augmented Dickey–Fuller (ADF) test tests the null hypothesis that a unit root is present in a time series sample. It makes use of the fact that if a price series has a mean-reverting behaviour, the next price level will then be proportional to the current price level. ADF test will be applied to each cluster and find out pairs that possess cointegration. Results include 21 pairs of stock and 30 unique tickers:

```
[(Equity(1665 [CMS]), Equity(2330 [DTE])),
 (Equity(2330 [DTE]), Equity(5484 [ES])),
 (Equity(7369 [TE]), Equity(7759 [UIL])),
 (Equity(7369 [TE]), Equity(8264 [TEG])),
 (Equity(7369 [TE]), Equity(8265 [WR])),
 (Equity(7369 [TE]), Equity(21458 [VVC])),
 (Equity(7369 [TE]), Equity(21975 [ALE])),
 (Equity(7369 [TE]), Equity(23126 [GXP])),
 (Equity(7369 [TE]), Equity(24852 [EE])),
 (Equity(7369 [TE]), Equity(25265 [AVA])),
 (Equity(7369 [TE]), Equity(26769 [NWE])),
 (Equity(7369 [TE]), Equity(28318 [POR])),
 (Equity(10639 [MAA]), Equity(11654 [HME])),
 (Equity(3488 [WELL]), Equity(18821 [VTR])),
 (Equity(3010 [FRT]), Equity(10528 [SPG])),
 (Equity(4238 [KIM]), Equity(8468 [SITC])),
 (Equity(7715 [UDR]), Equity(10528 [SPG])),
 (Equity(7715 [UDR]), Equity(11465 [ESS])),
 (Equity(122 [ADI]), Equity(4485 [LLTC])),
 (Equity(510 [ARG]), Equity(6272 [PX])),
 (Equity(6583 [RTN]), Equity(12691 [LMT]))]
```

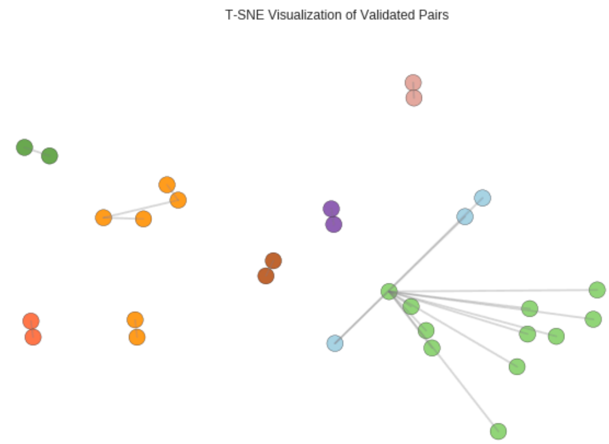

FIGURE 5 – THE 21 PAIRS

FIGURE 6 – T-SNE OF VALIDATED PAIRS

The procedure successfully screened out 21 ADF-validated pairs from more than 2 millions candidates. Out of 21 pairs, 1 pair from the semiconductor sector, 1 pair from the energy sector, 1 pair from real estate investment trust (REIT) and 1 pair from the aerospace and defense (A&D) sector are chosen to be back-tested in the following of the report, including Raytheon Company and Lockheed Martin Corp, DTE Energy Co and CMS Energy Corporation, Kimco Realty Corp and Site Centers Corp, and Analog Devices Inc and Linear Technology.

Raytheon Company and Lockheed Martin Corp

FIGURE 7 – PRICE CHART OF RTN AND LMT

Raytheon Company (RTN) and Lockheed Martin Corporation (LMT) are companies that specialize in defense and other government markets. RTN develops integrated products, services and solutions in various markets, including sensing; effects; command, control, communications, computers, cyber and intelligence; mission support, and cybersecurity [3]. LMT specializes in the research, design, development, manufacture, integration and sustainment of technology systems, products and services [4].

DTE Energy Co and CMS Energy Corporation

FIGURE 8 – PRICE CHART OF DTE AND CMS

DTE Energy Company (DTE) is an energy company. The Company's segments include Electric, Gas, Gas Storage and Pipelines, Power and Industrial Projects, Energy Trading, and Corporate and Other [5]. CMS Energy Corporation (CMS Energy), incorporated on February 26, 1987, is also an energy company. The Company operates through three segments: electric utility, gas utility, and enterprises [6]. Both are US energy companies primarily operating in Michigan, giving them a solid ground for fundamentals on pair trading.

Kimco Realty Corp and Site Centers Corp

FIGURE 9 – PRICE CHART OF KIM AND SITC

Kimco Realty Corporation (KIM) and Centres Corp. (SITC) are self-administered REIT. KIM is engaged in the ownership, management, development and operation of open-air shopping centres, which are anchored generally by discount department stores, grocery stores or drugstores [7]. SITE is in the business of acquiring, owning, developing, redeveloping, expanding, leasing and managing shopping centres [8]. The two REIT share similar fundamentals factors as they have a similar revenue stream and business model, making them a potentially profitable pair.

Analog Devices Inc and Linear Technology Corp

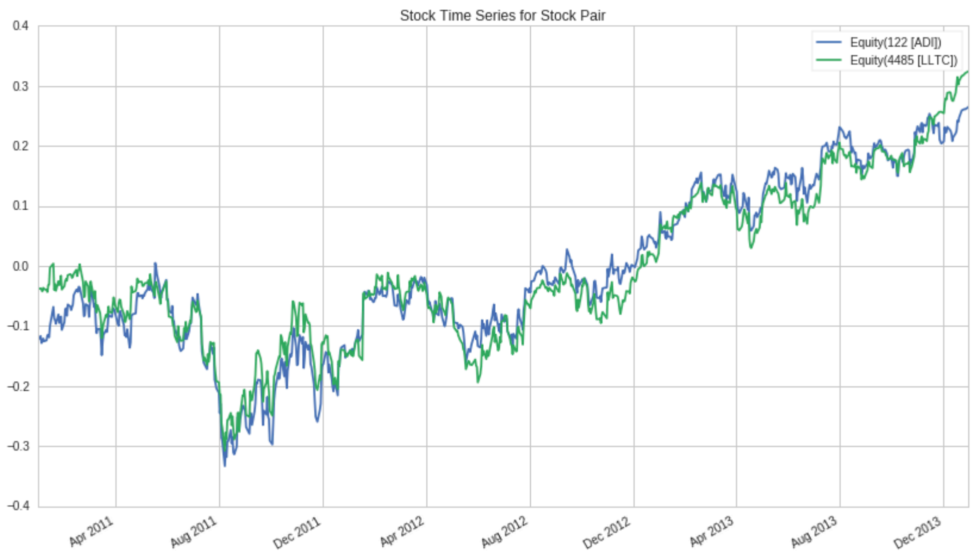

FIGURE 10 – PRICE CHART OF ADI AND LLTC

Analog Devices Inc (ADI) is a semiconductor manufacturing company that designs, manufactures and markets a portfolio of solutions that leverage high-performance analog, mixed-signal and digital signal processing(DSP) technology, including integrated circuits (ICs), algorithms, software and subsystems [9]. Linear Technology Corporation (LLTC), acquired by ADI in 2017, designs, manufactures, and markets a line of linear integrated circuits. The Company's products include high-performance amplifiers, comparators, voltage references, monolithic filters, linear regulators, DC-DC converters, battery chargers, data converters, communications interface circuits, and RF signal conditioning circuits [10].

Strategy Implementation

Statistical Arbitrage

Statistical arbitrage exploits the pricing inefficiency between mean-reverting pairs of assets or buckets of assets in a market. The term mean-reversion means the assumption that the stock price tends to converge to the mean price over time. Therefore, the discrepancy between the stock price and the average price will diverge and converge in time. Hence, statistical arbitrage strategy makes profit from the spread; for example, one can long/short a stock which has a price lower than expected because when the gap narrows, profit can be generated from this spread. A statistical arbitrage model about two assets class can be expressed as follows [11]:

$$\frac{dP_t}{P_t} = \alpha dt + \beta \frac{dQ_t}{Q_t} + dXt,$$

where P_t and Q_t are the respective time series for assets P and Q and X_t is a mean-reverting process. X_t determines the trading direction of the pair: long asset P and short β dollars of stock Q for \$1-worth asset P if X_t is small, vice versa.

When the cointegration or mean-reverting relationship can break down over time, the strategy will, consequently, suffer and black swan, a low probability market movement, may impose heavy short-term losses. To keep track of this relationship throughout the back-testing, several tests are used to constantly evaluate the behaviours of the pair, including ADF test as described before, Hurst exponent that helps identify whether a time series is trending, mean-reverting or undergoing a random walk, half-life test from the Ornstein–Uhlenbeck (OU) Process that tests for the mean-reversion speed.

Hurst Exponent

Hurst exponent relates to the autocorrelations of the time series, and the rate at which these decrease as the lag between pairs of values increases [12]. The Hurst exponent studies were originated from hydrology. It is used to provide a scalar value that identifies if a time series is trending, mean-reverting or undergoing Brownian motion.

$$\begin{cases} \mathrm{H} < 0.5, \text{the time series is trending} \\ \mathrm{H} = 0.5, \text{the time series is undergoing Brownian motion} \\ \mathrm{H} > 0.5, \text{the time series is trending} \end{cases}$$

Half-life Test

Ornstein–Uhlenbeck (OU) process x_t is a mean-reverting process defined by the stochastic differential equation:

$$dx_t = -\theta\, x_t\, dt + \sigma\, dW_t$$

where X_t is a Brownian motion with unit variance parameter and θ and σ are constants. θ, the speed of mean reversion, can be used to find the half-life of the mean-reverting process:

$$t_{1/2} = \frac{log2}{\theta}$$

A time series shows mean-reversion unnecessarily means that one can profit from trading it in a particular time horizon. Thus, a half-life threshold will be set and any time series observed with a half-life longer than pre-set value will be disregarded.

Kalman Filter and Spread – A Mean Deviation Measurement

For a stationary and cointegrated series, a spread between the pair of assets, which is essentially a difference between their prices, is assumed to fluctuate around a mean value. The spread can be calculated by finding slope, usually called hedge ratio, of a linear regression between two time series. Rather than using OLS regression, Kalman filter, a state space model, will be used to provide a dynamic hedge ratio without giving a specific regression window length and with a presence of statistical noise.

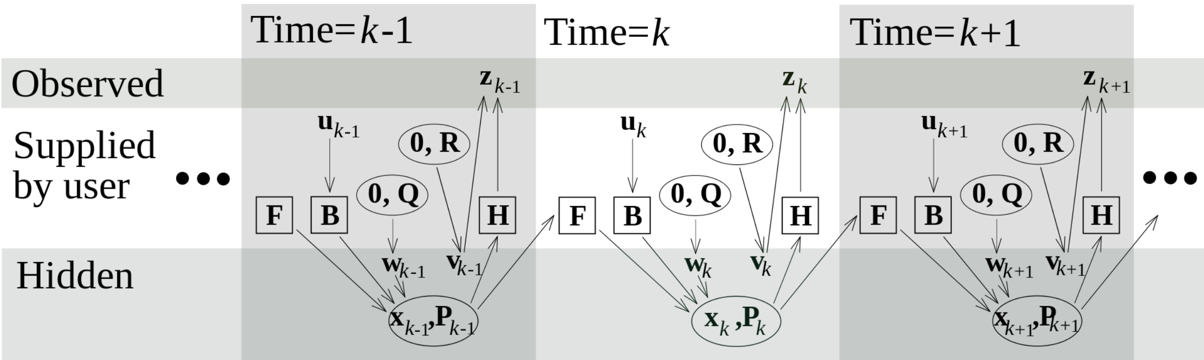

FIGURE 11 – WORKING MECHANISM OF KALMAN FILTER

The Kalman filter works recursively by two steps: in the prediction step, it produces estimates of the current state variables, along with their current state variance; in the updating step, these estimates are updated using a weighted average, with higher weighting given to estimates with higher certainty to deal with measurement error or random noises after the next measurement is observed. A standard score of the spread will be calculated accordingly and can be used to generate trading signals.

Trading Rules

Trading Signal

126 trading days will be the window of observation. A trading signal will be generated if the time series passes either one of the three tests. The trade will be positioned if the absolute value of spread standard score is larger than the open threshold and be closed as long as the standard score reaches the close threshold. Here are the parameters used in the back-testing:

Controls	
Time Frame	Daily
Initial Capital	\$100000
Max. Net Leverage	1.0
Benchmark	SPDR S&P 500 ETF
Trading Signals	
Parameters	Value
Open Z score	1.0
Close Z score	0.0
Hurst Exponent	
Window Length	126 (Days)
Upper Threshold	0.4
Lower Threshold	0
Lag	100
Half-life Test	
Max. Duration	43 (Days)
Min. Duration	43 (Days)
ADF Test	
Window	63(Days)
Max. P-value	0.05
Kalman Filter	
Initial State Mean	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
Initial State Variance	$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
Delta	1^{-5}
Transition Matrices	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Transition Covariance	$1^{-5}/(1-1^{-5})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Observation Matrices	$[P_t \;\; 1]$
Observation Covariance	1.0

TABLE 1 – PARAMETERS FOR BACK-TESTING

Market Frictions

To make the results more meaningful, market frictions, e.g. slippage and transaction costs, are also considered. The slippage is a percentage of 0.05%, and multiplying that percentage by the order price. Buys will fill at a price that is 0.05% higher than the close of the next minute, while sells will fill at a price that is 0.05% lower; vice versa. As a position of pair trading comprises two stocks, it is sensible to include commissions. In the back-testing, \$5 per share trade (\$10 per position) is charged.

Results Interpretation

Performance Evaluation

The following three-year bask-tests run daily from 1st January 2014 to 31st December 2016 in alignment with the stock selection dated on 31st December 2013. This is the performance summary of the selected four pairs:

	Benchmark	RTN/LMT	DTE/CMS	KIM/SITC	ADI/LLTC	Pairs Average
Annual return	8.7%	18.2%	18.9%	7.2%	11.2%	13.88%
Cumulative returns	28.4%	65.0%	67.8%	23.1%	37.2%	48.28%
Annual volatility	13.1%	12.8%	13.9%	14.4%	23.7%	16.20%
Sharpe ratio	0.7	1.37	1.31	0.55	0.56	0.95
Max drawdown	-12.4%	-10.2%	-12.4%	-24.2%	-18.2%	-16.25%
Alpha	0	0.14	0.15	0.03	0.07	0.098
Beta	0.98	0.36	0.34	0.51	0.65	0.465

TABLE 2 – SUMMARY OF RESULTS

Table 2 shows that trading pairs are on average more profitable (13.88% vs 8.70%) than the benchmark, S&P 500 ETF. While RTN/LMT, DTE/CMS, and ADI/LLTC generate considerable returns from 37.2% to 67.8%, KIM/SITC failed to beat the benchmark with its 23.1% cumulative returns in three years. All pairs successfully generate alpha with a relatively low beta and a higher Sharpe Ratio. However, pairs except RTN/LMT suffer from a higher maximum drawdown and higher volatility than the benchmark.

Pair RTN/LMT

FIGURE 12 – CUMULATIVE RETURNS OF RTN/LMT

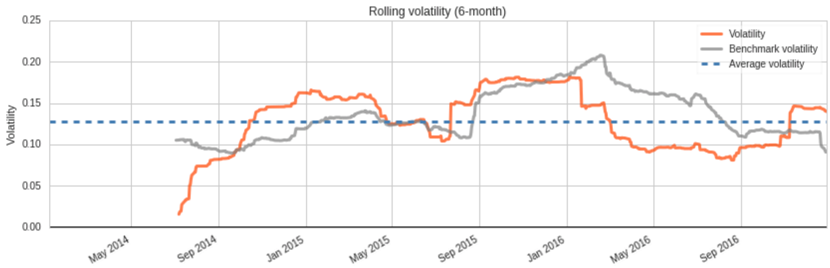

FIGURE 13 – ROLLING VOLATILITY OF RTN/LMT

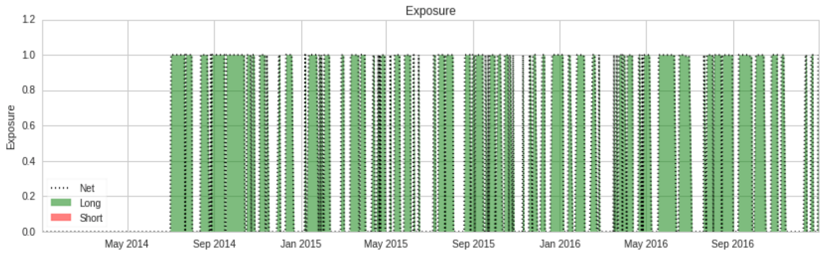

FIGURE 14 – MARKET EXPOSURE OF RTN/LMT

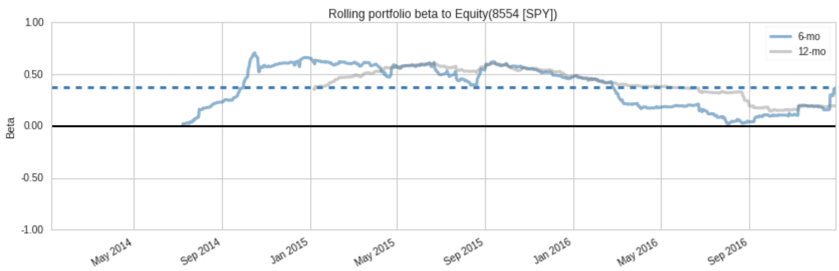

FIGURE 15 – ROLLING BETA OF RTN/LMT

Figure 12 and Figure 13 show that the pair beat the market since Q2 2015 with a lower volatility on average (-0.3%). Figure 14 presents a low market exposure. However, the beta is relatively low but not market neutral over time as shown in Figure 15 while alpha is generated.

Pair DTE/CMS

FIGURE 16 – CUMULATIVE RETURNS OF DTE/CMS

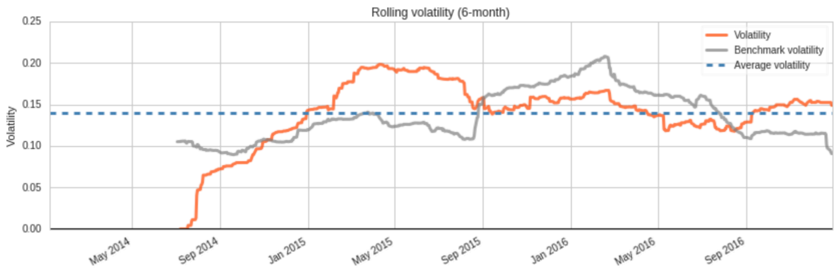

FIGURE 17– ROLLING VOLATILITY OF DTE/CMS

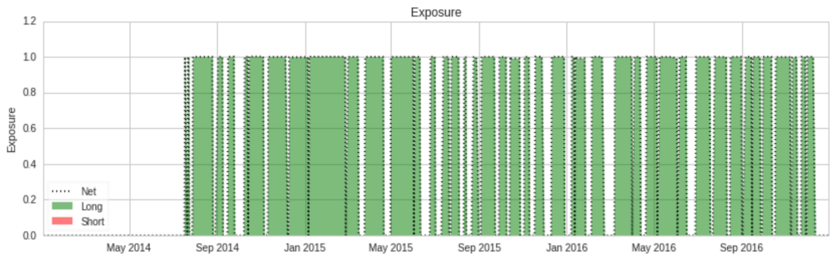

FIGURE 18 - MARKET EXPOSURE OF DTE/CMS

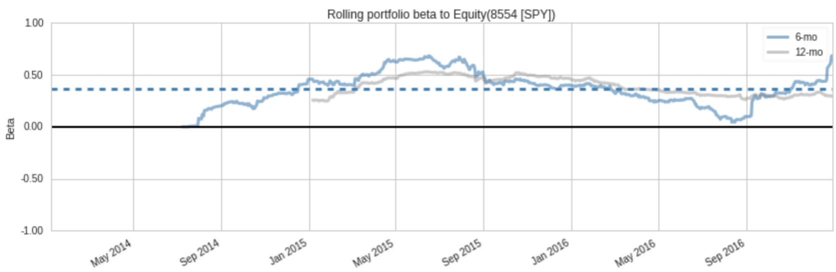

FIGURE 19 – ROLLING BETA OF DTE/CMS

Figure 16 and Figure 17 show that the pair outperformed the market most of the time with a only 0.8% higher annual volatility. Figure 18 presents that market exposure is medium. Again, the beta is relatively low but not market neutral over time as shown in Figure 19 while alpha is generated.

Pair KIM/SITC

FIGURE 20 – CUMULATIVE RETURNS OF KIM/SITC

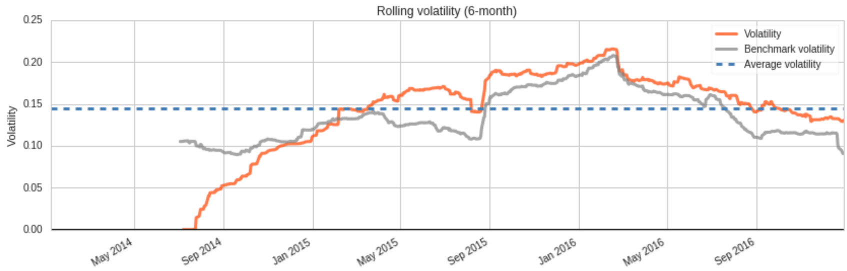

FIGURE 21– ROLLING VOLATILITY OF KIM/SITC

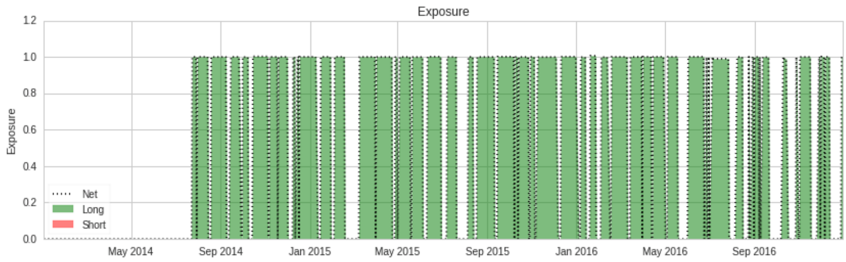

FIGURE 22 – MARKET EXPOSURE OF KIM/SITC

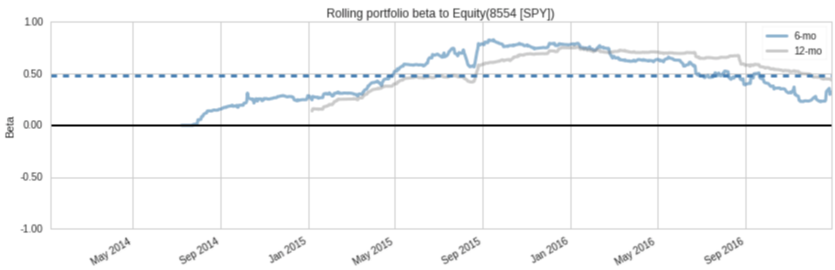

FIGURE 23 - ROLLING BETA OF KIM/SITC

Figure 20 and Figure 21 show that the pair underperformed the market with a higher average volatility. Figure 22 presents a medium market exposure. Again, the beta is relatively low but not market neutral over time as shown in Figure 23 and a small alpha (0.03) is generated.

Pair ADI/LLTC

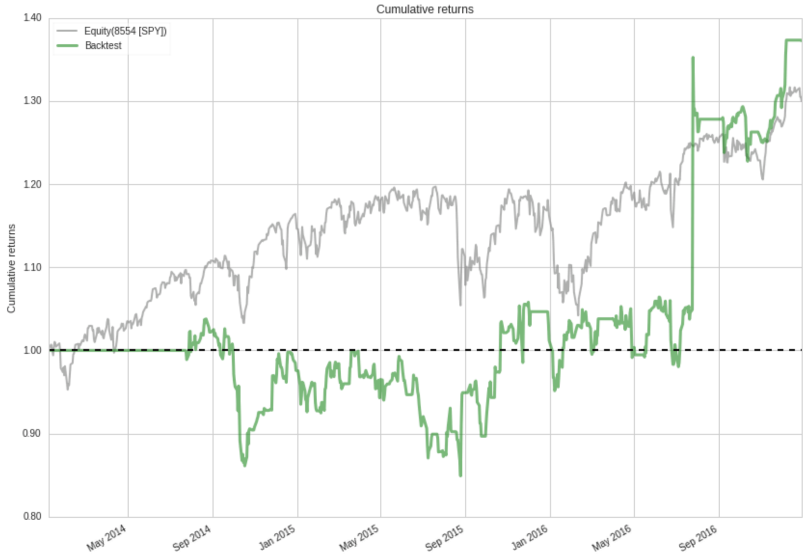

FIGURE 24 – CUMULATIVE RETURNS OF ADI/LLTC

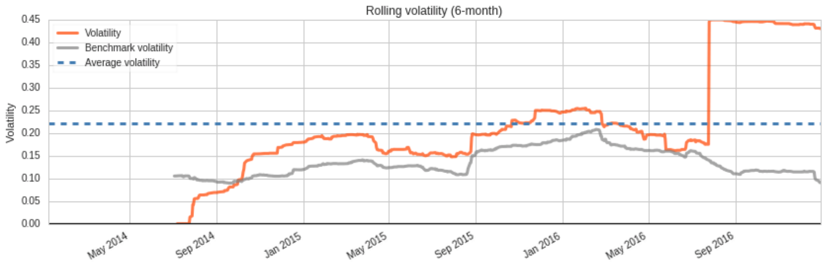

FIGURE 25 - ROLLING VOLATILITY OF ADI/LLTC

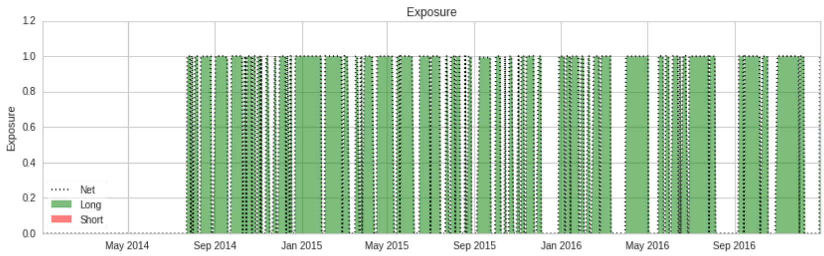

FIGURE 26 - MARKET EXPOSURE OF ADI/LLTC

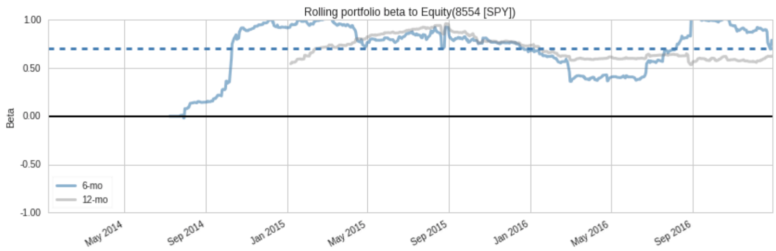

FIGURE 27 - ROLLING BETA OF ADI/LLTC

Figure 24 and Figure 25 show that the pair underperformed the market until Q3 2016 with a high volatility. Figure 26 presents that market exposure is relatively low. And, the beta is completely not market neutral over time as shown in Figure 27 while alpha is generated.

Concluding Remarks

This report studied a viable way to pick stocks from the vast pool to be used in pairs trading, trade within the US equity market and profit from the strategy. Incorporating PCA and DBSCAN in the selection process of the stock, millions of pairs of stocks that are statistically determined to be non-stationary. Hedge ratio is crucial in long/shorting the two stocks, as it is used to calculate the spread so as to generate a profitable trading signal. Kalman filter is used here to calculate the dynamic hedge ratio other than linear regression. The results of the back-testing that utilizes ADF, Hurst exponent are promising, as the strategy successfully demonstrated alpha with a relatively low but yet neutral beta.

Future Work

More Robust Trading Algorithm

The initial validation window costs 126 trading days. As a result, all pairs trade only after July 2014, limiting the profitability since the pairs were believed to be cointegrated on 31 December 2013 and the relationship can break down in a short time. In the future, the algorithm can be extended such that it could validate the pair at the start of back-testing.

Parameters Optimization

Trading parameters; e.g. the open and close spread standard score, are not optimized for each pair. The spread standard score favourable in a long position does not always apply to the short counterpart. Optimization of portfolio weighting and leverage can also be implemented to achieve better performance.

More Fundamental Factors

In the pair selection process, only fundamental factors like market cap and financial health grade are considered. Future studies can potentially add up to more common factors like earnings momentum to filter stocks. And the US equity universe can be extended to more than 3000 stocks.

Other Clustering and Selecting Methods

The pairs selected DBSCAN were almost all coming from the same sector. While a good pair can be a pair of stocks across sectors, other clustering algorithms like model-based clustering and fuzzy clustering can be used. Besides, pairs are only verified by the ADF test. The tickers that were not statistically determined to be stationary are therefore removed from the stock pool. In the future, the OU process can be adopted to account for non-stationary stocks, then more profitable pairs can be discovered.

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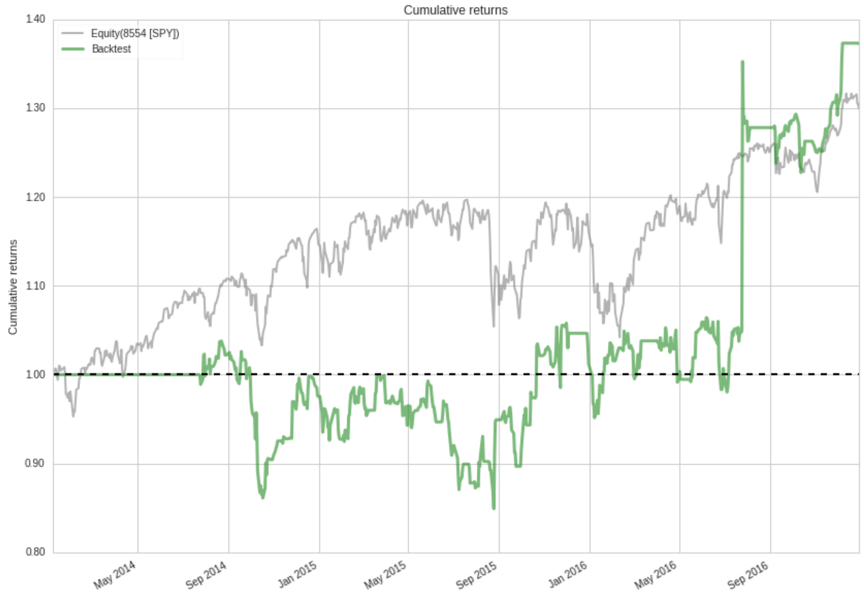
Appendix

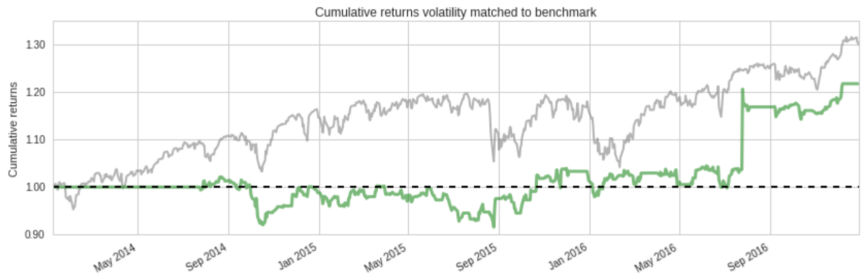
Full Tear Sheet of Pair ADI/LLTC

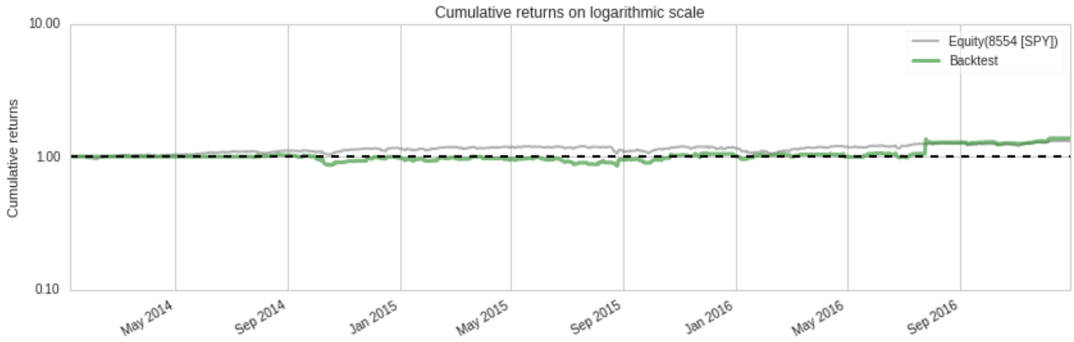
Start date	2014-01-06
End date	2016-12-30
Total months	35

	Backtest
Annual return	11.2%
Cumulative returns	37.2%
Annual volatility	23.7%
Sharpe ratio	0.56
Calmar ratio	0.61
Stability	0.38
Max drawdown	-18.2%
Omega ratio	1.19
Sortino ratio	1.17
Skew	9.86
Kurtosis	186.67
Tail ratio	1.02
Daily value at risk	-2.9%
Gross leverage	0.51
Daily turnover	32.0%
Alpha	0.07
Beta	0.65

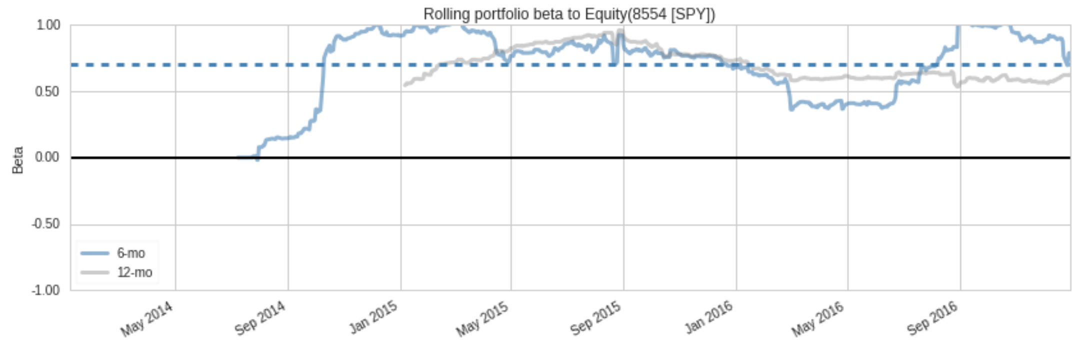
Worst drawdown periods	Net drawdown in %	Peak date	Valley date	Recovery date	Duration
0	18.19	2014-08-22	2015-08-25	2015-11-18	324
1	10.07	2015-12-01	2016-01-08	2016-06-01	132
2	9.24	2016-07-26	2016-10-13	2016-12-07	97
3	7.88	2016-06-07	2016-07-05	2016-07-26	36
4	6.48	2015-11-18	2015-11-23	2015-11-25	6

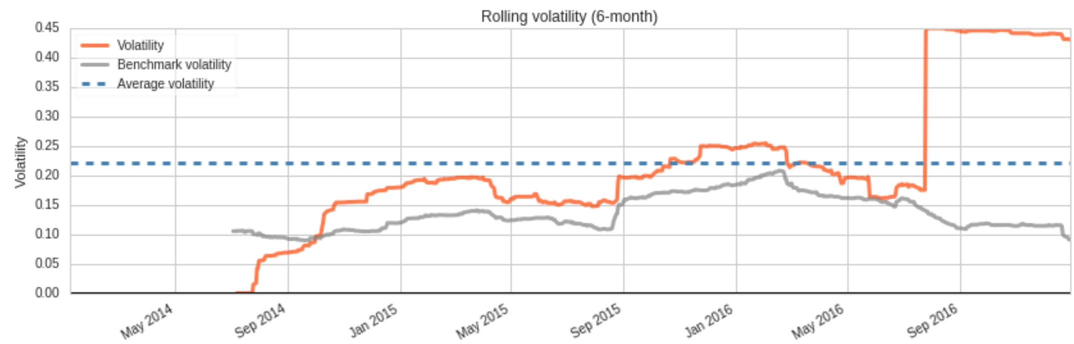


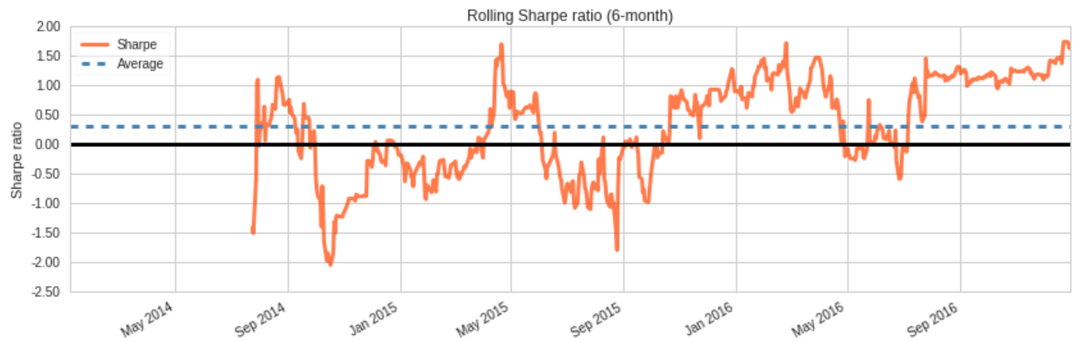


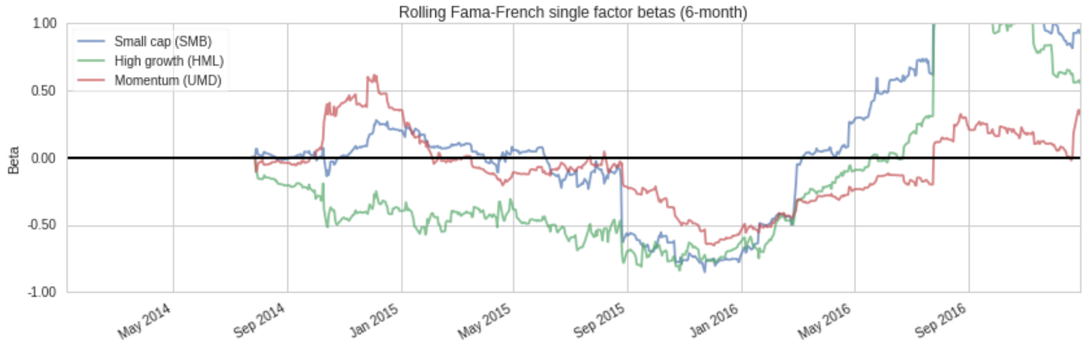

Cumulative returns on logarithmic scale
Cumulative returns
10.00
1.00
0.10
Equity(8554 [SPY])
Backtest
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

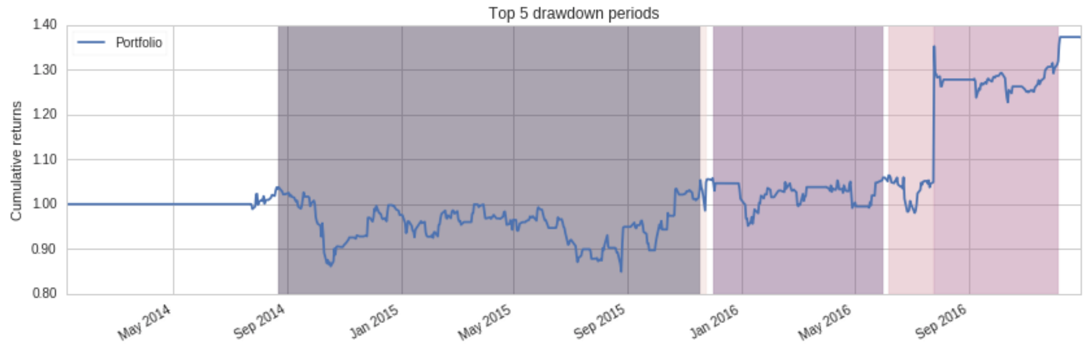

Returns
Returns
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

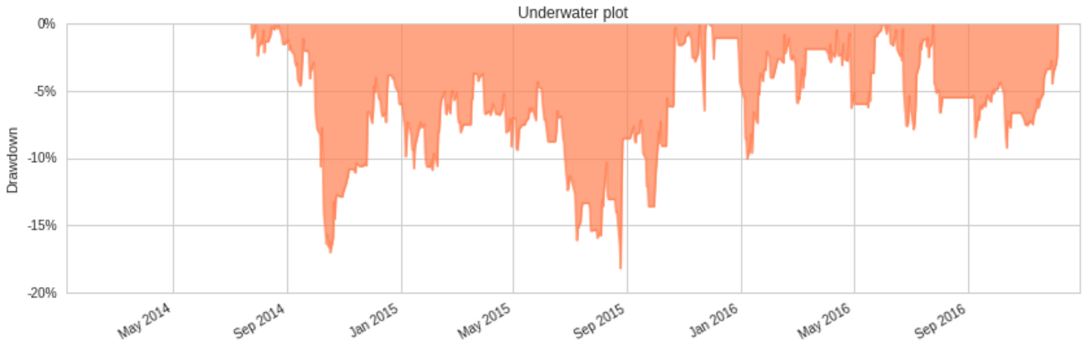

Rolling portfolio beta to Equity(8554 [SPY])
Beta
1.00
0.50
0.00
-0.50
-1.00
6-mo
12-mo
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

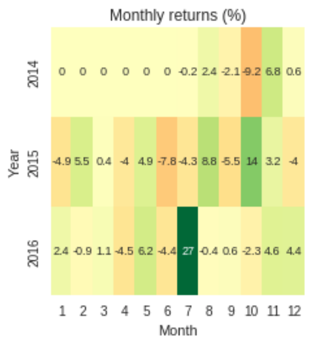

Rolling volatility (6-month)
Volatility
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Volatility
Benchmark volatility
Average volatility
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016


Rolling Sharpe ratio (6-month)
Sharpe ratio
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
Sharpe
Average
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

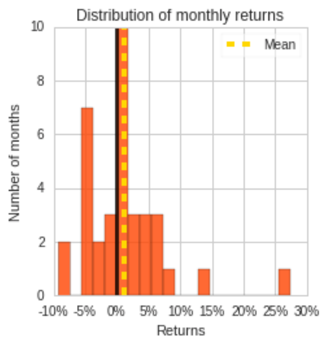

Rolling Fama-French single factor betas (6-month)
Small cap (SMB)
High growth (HML)
Momentum (UMD)
Beta
1.00
0.50
0.00
-0.50
-1.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

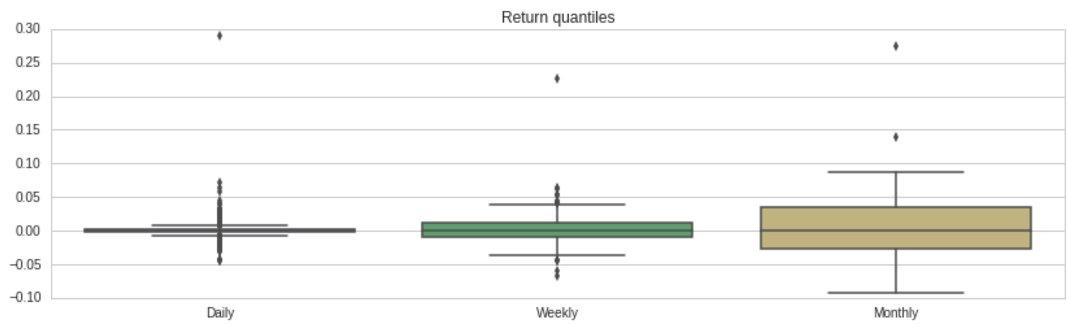

Top 5 drawdown periods
Portfolio
Cumulative returns
1.40
1.30
1.20
1.10
1.00
0.90
0.80
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016


Underwater plot
Drawdown
0%
-5%
-10%
-15%
-20%
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

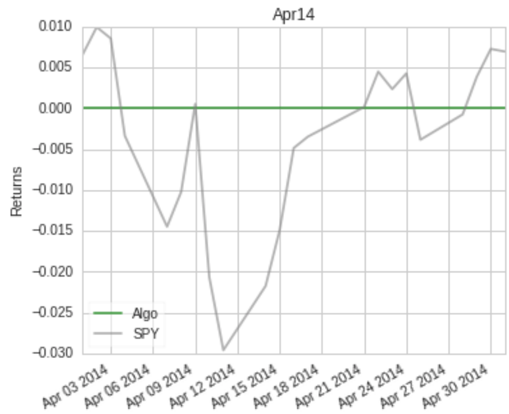

Monthly returns (%)
Year
2014
2015
2016
0 0 0 0 0 0 -0.2 2.4 -2.1 -9.2 6.8 0.6
-4.9 5.5 0.4 -4 4.9 -7.8 -4.3 8.8 -5.5 14 3.2 -4
2.4 -0.9 1.1 -4.5 6.2 -4.4 27 -0.4 0.6 -2.3 4.6 4.4
1 2 3 4 5 6 7 8 9 10 11 12
Month

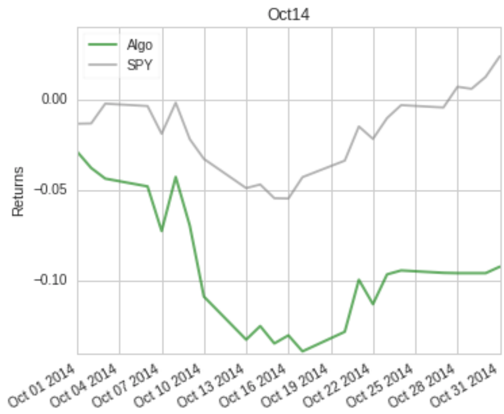

Annual returns
Mean
Year
2014
2015
2016
-5% 0% 5% 10% 15% 20% 25% 30% 35% 40%
Returns

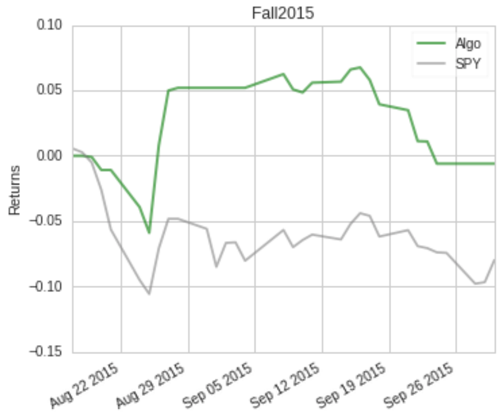

Distribution of monthly returns
Mean
Number of months
10
8
6
4
2
0
-10% -5% 0% 5% 10% 15% 20% 25% 30%
Returns


Return quantiles
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
Daily
Weekly
Monthly

Stress Events	mean	min	max
Apr14	0.00%	0.00%	0.00%
Oct14	-0.40%	-4.20%	3.29%
Fall2015	-0.00%	-2.87%	7.12%
New Normal	0.05%	-4.50%	28.92%





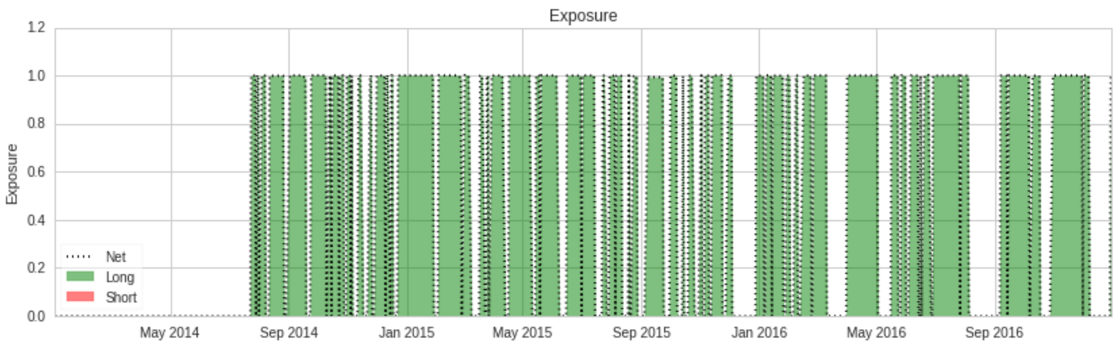


Top 10 long positions of all time	max
LLTC-4485	100.20%
ADI-122	100.18%

Top 10 short positions of all time	max

Top 10 positions of all time	max
LLTC-4485	100.20%
ADI-122	100.18%

All positions ever held	max
LLTC-4485	100.20%
ADI-122	100.18%

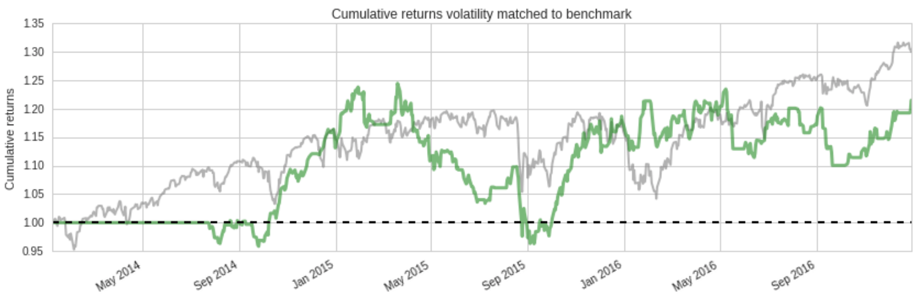


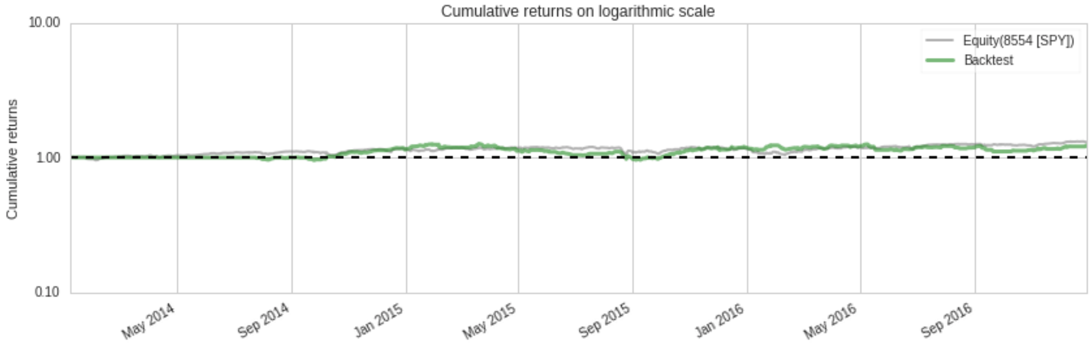
Full Tear Sheet of Pair KIM/SITC

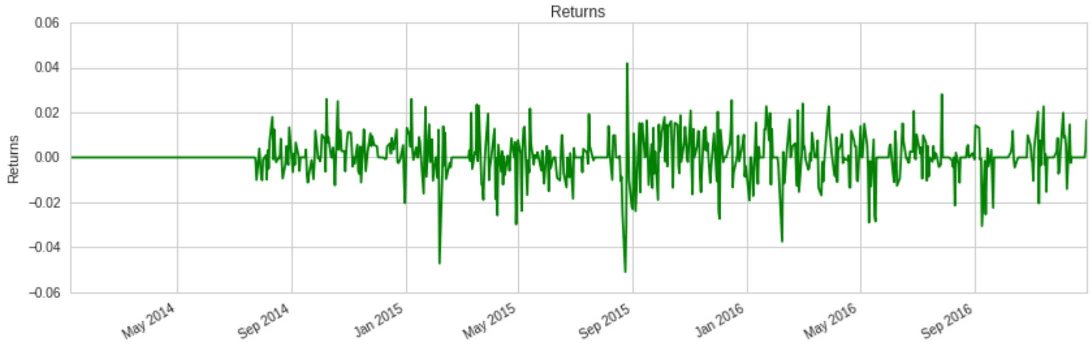
Start date	2014-01-06
End date	2016-12-30
Total months	35

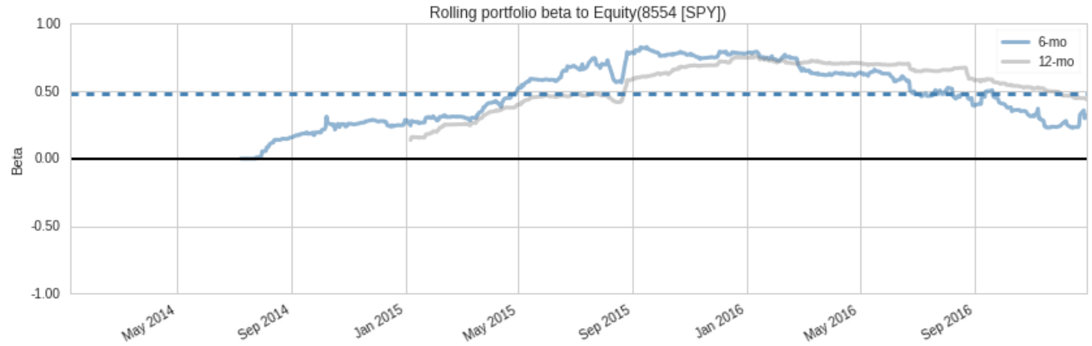
	Backtest
Annual return	7.2%
Cumulative returns	23.1%
Annual volatility	14.4%
Sharpe ratio	0.55
Calmar ratio	0.30
Stability	0.47
Max drawdown	-24.2%
Omega ratio	1.12
Sortino ratio	0.77
Skew	-0.52
Kurtosis	4.14
Tail ratio	1.09
Daily value at risk	-1.8%
Gross leverage	0.58
Daily turnover	27.8%
Alpha	0.03
Beta	0.51

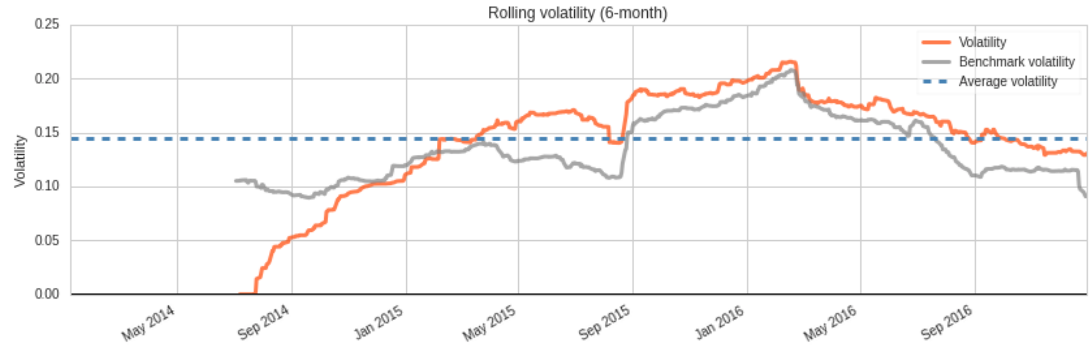
Worst drawdown periods	Net drawdown in %	Peak date	Valley date	Recovery date	Duration
0	24.20	2015-03-20	2015-09-09	NaT	NaN
1	6.15	2015-01-29	2015-02-09	2015-03-20	37
2	4.85	2014-08-29	2014-09-25	2014-10-09	30
3	3.95	2014-07-23	2014-08-07	2014-08-20	21
4	3.01	2014-12-29	2014-12-31	2015-01-07	8

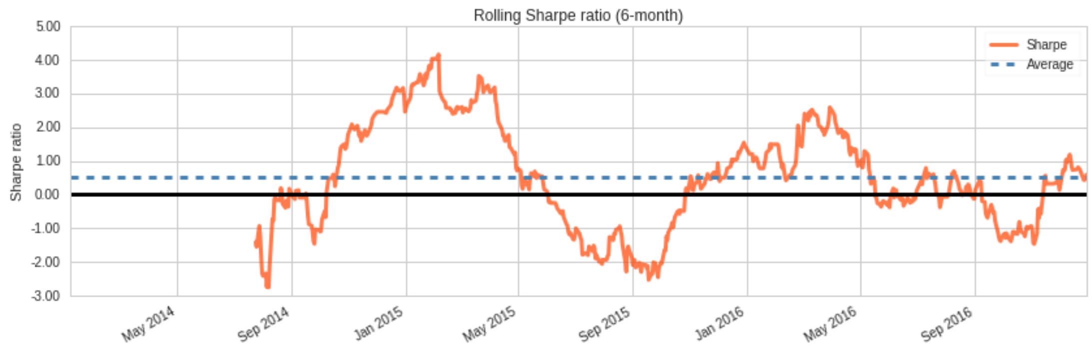


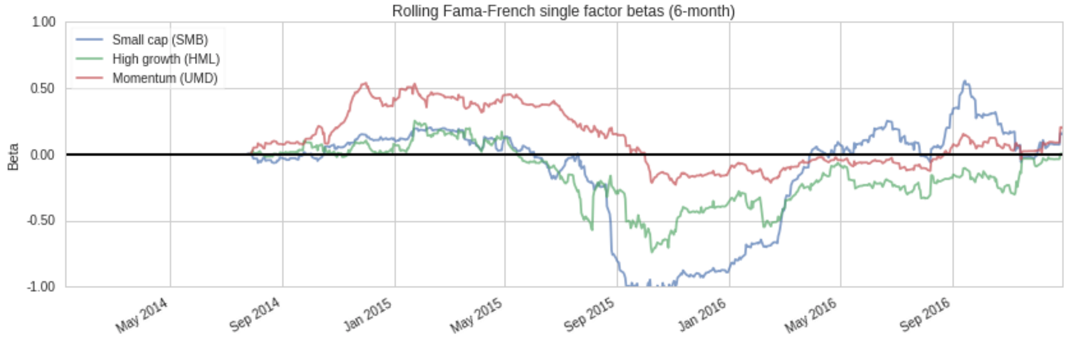

Cumulative returns on logarithmic scale
Cumulative returns
10.00
1.00
0.10
Equity(8554 [SPY])
Backtest
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

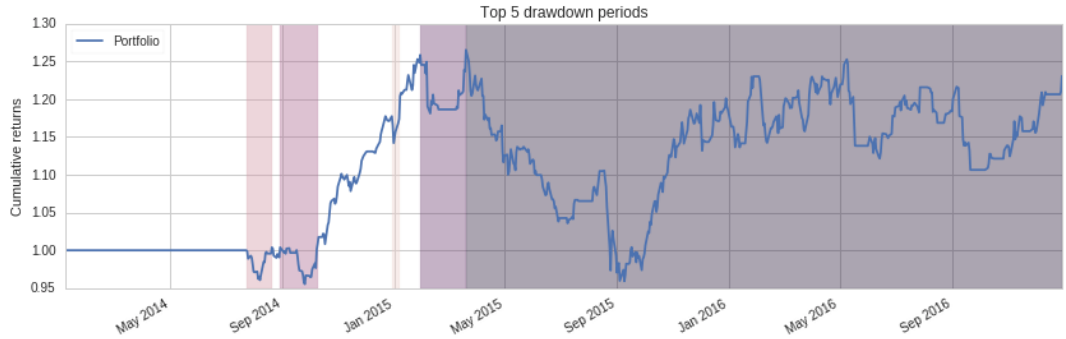

Returns
Returns
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

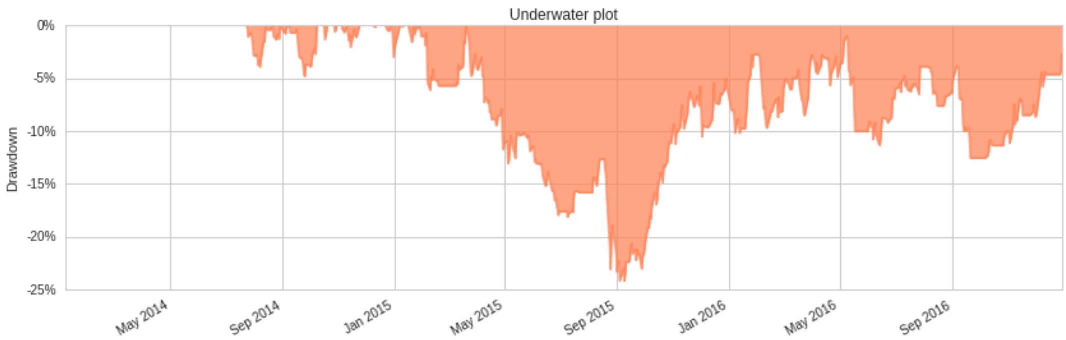

Rolling portfolio beta to Equity(8554 [SPY])
Beta
1.00
0.50
0.00
-0.50
-1.00
6-mo
12-mo
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

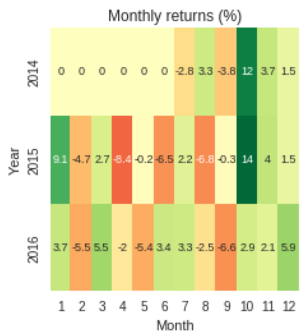

Rolling volatility (6-month)
Volatility
0.25
0.20
0.15
0.10
0.05
0.00
Volatility
Benchmark volatility
Average volatility
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

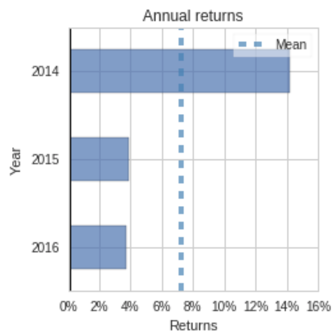

Rolling Sharpe ratio (6-month)
Sharpe ratio
5.00
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
Sharpe
Average
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

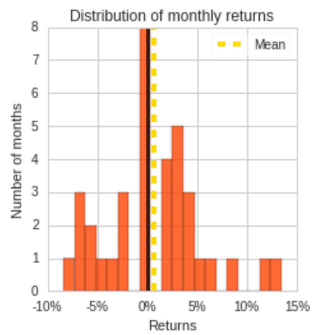

Rolling Fama-French single factor betas (6-month)
Small cap (SMB)
High growth (HML)
Momentum (UMD)
Beta
1.00
0.50
0.00
-0.50
-1.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

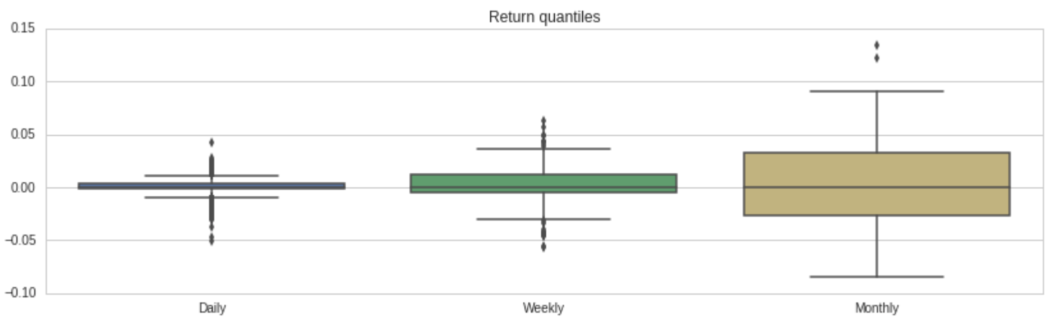

Top 5 drawdown periods
Portfolio
Cumulative returns
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016


Underwater plot
Drawdown
0%
-5%
-10%
-15%
-20%
-25%
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

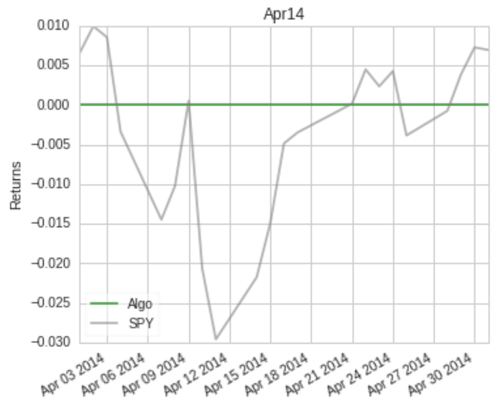

Monthly returns (%)
Year
2014
0 0 0 0 0 0 -2.8 3.3 -3.8 12 3.7 1.5
2015
9.1 -4.7 2.7 -8.4 -0.2 -6.5 2.2 -5.8 -0.3 14 4 1.5
2016
3.7 -5.5 5.5 -2 -5.4 3.4 3.3 -2.5 -6.6 2.9 2.1 5.9
1 2 3 4 5 6 7 8 9 10 11 12
Month

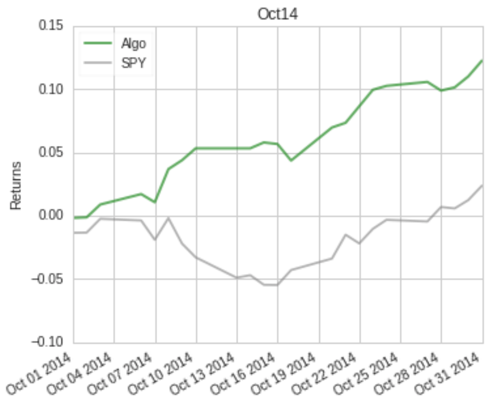

Annual returns
Mean
Year
2014
2015
2016
0% 2% 4% 6% 8% 10% 12% 14% 16%
Returns

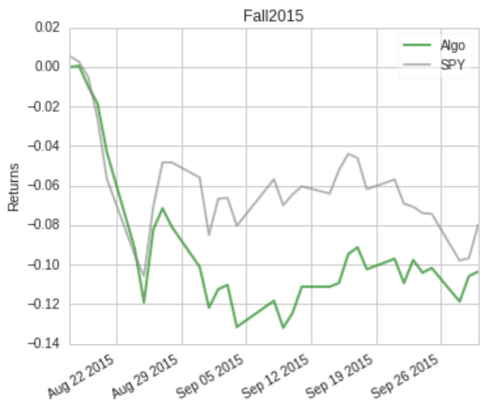

Distribution of monthly returns
Mean
Number of months
8
7
6
5
4
3
2
1
0
-10% -5% 0% 5% 10% 15%
Returns

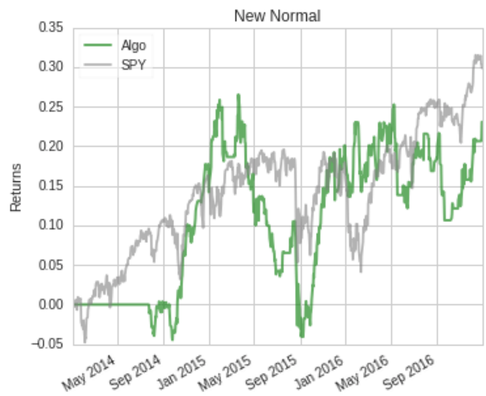

Return quantiles
0.15
0.10
0.05
0.00
-0.05
-0.10
Daily
Weekly
Monthly

Stress Events	mean	min	max
Apr14	0.00%	0.00%	0.00%
Oct14	0.51%	-1.23%	2.59%
Fall2015	-0.33%	-5.09%	4.18%
New Normal	0.03%	-5.09%	4.18%







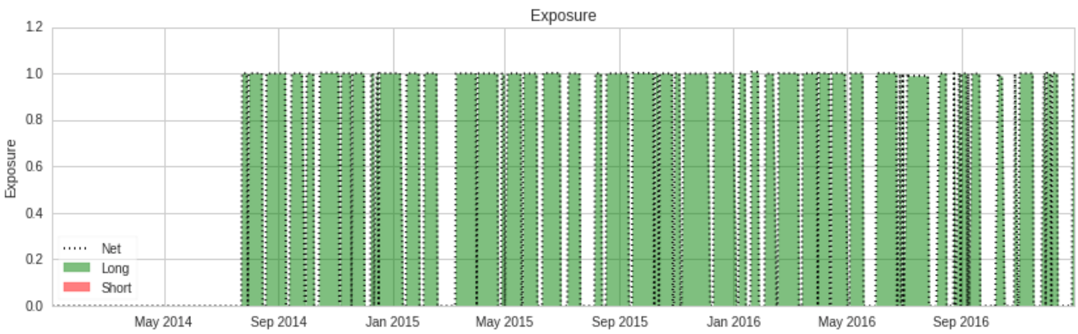


Top 10 long positions of all time	max
SITC-8468	100.65%
KIM-4238	100.16%

Top 10 short positions of all time	max

Top 10 positions of all time	max
SITC-8468	100.65%
KIM-4238	100.16%

All positions ever held	max
SITC-8468	100.65%
KIM-4238	100.16%

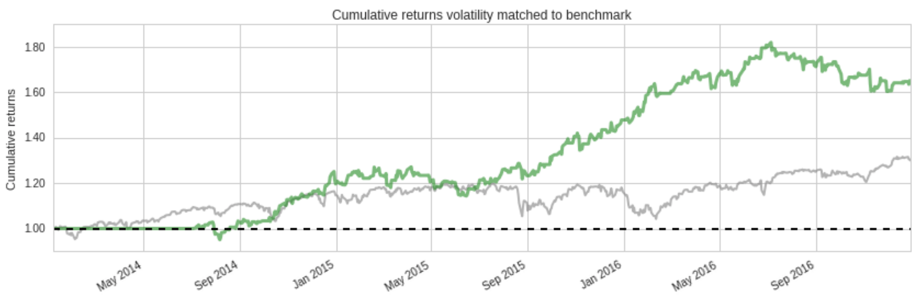


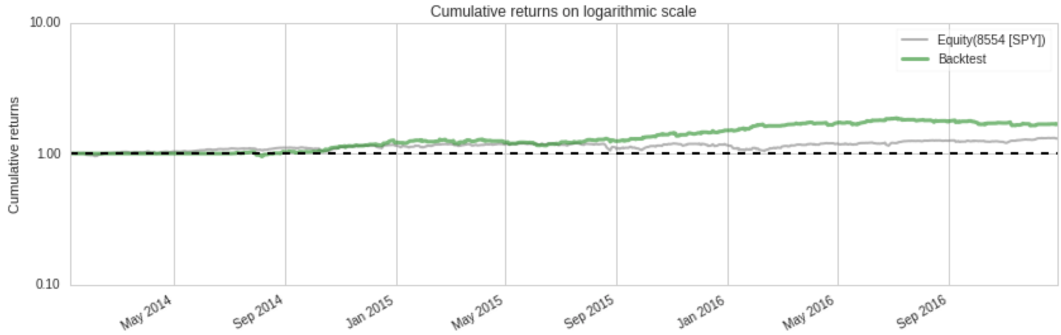
Full Tear Sheet of Pair CMS/DTE

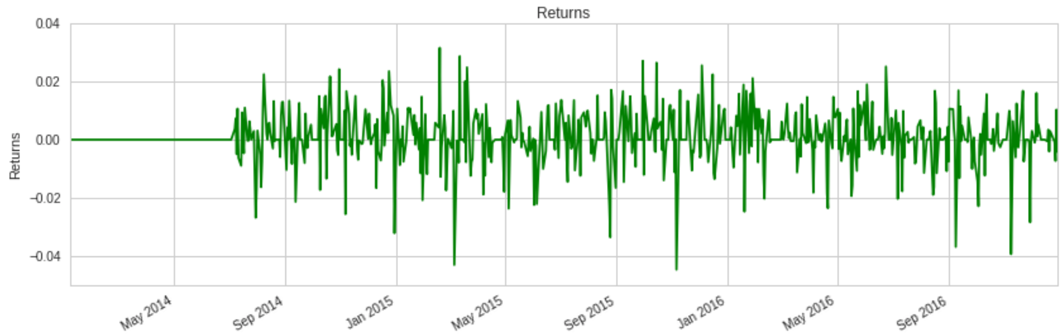
Start date	2014-01-06
End date	2016-12-30
Total months	35

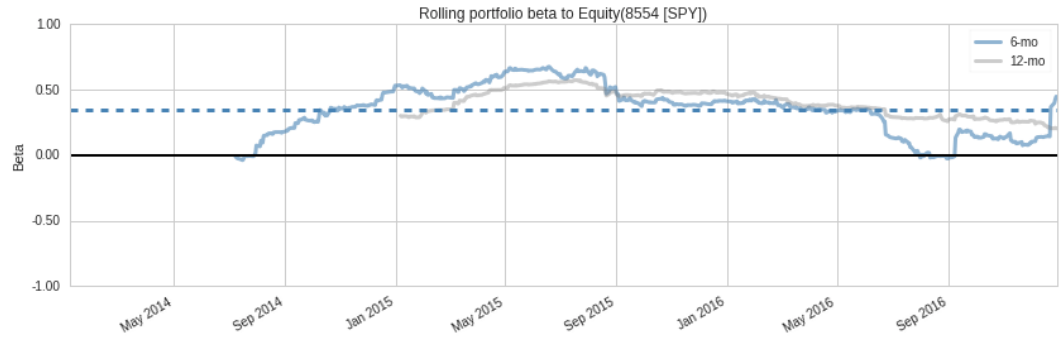
	Backtest
Annual return	18.9%
Cumulative returns	67.8%
Annual volatility	13.9%
Sharpe ratio	1.31
Calmar ratio	1.52
Stability	0.93
Max drawdown	-12.4%
Omega ratio	1.31
Sortino ratio	1.85
Skew	-0.69
Kurtosis	3.89
Tail ratio	1.03
Daily value at risk	-1.7%
Gross leverage	0.58
Daily turnover	35.7%
Alpha	0.15
Beta	0.34

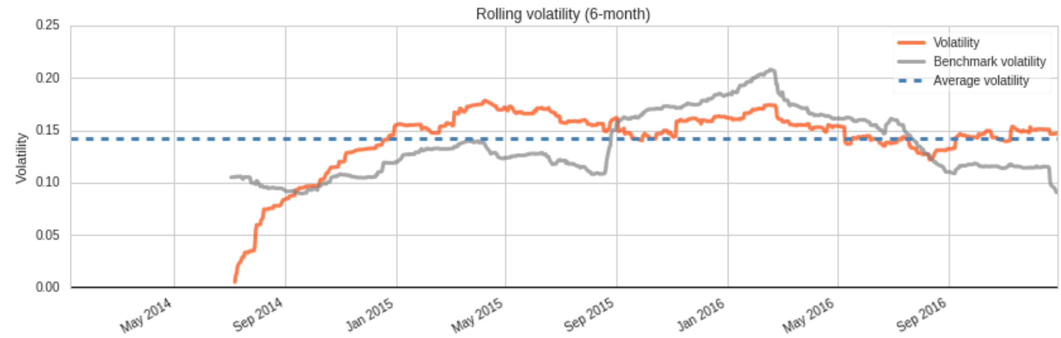
Worst drawdown periods	Net drawdown in %	Peak date	Valley date	Recovery date	Duration
0	12.38	2016-07-06	2016-12-01	NaT	NaN
1	10.50	2015-04-06	2015-06-15	2015-08-19	98
2	7.89	2014-07-28	2014-08-06	2014-08-29	25
3	7.41	2015-02-18	2015-03-11	2015-04-06	34
4	5.54	2015-08-21	2015-08-25	2015-09-30	29

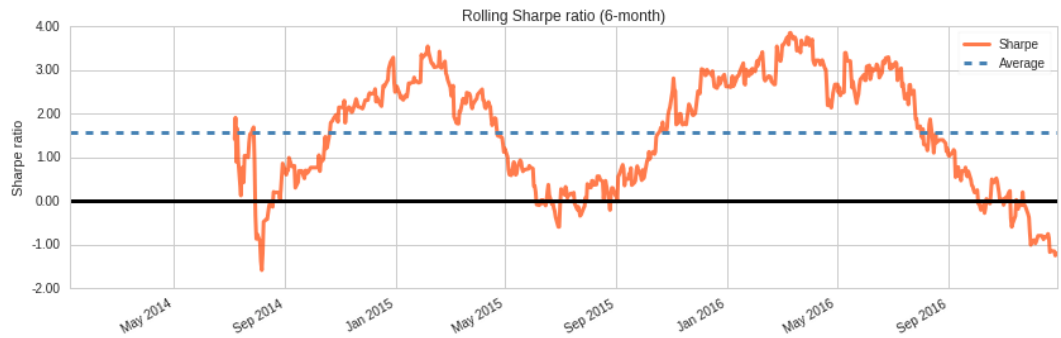


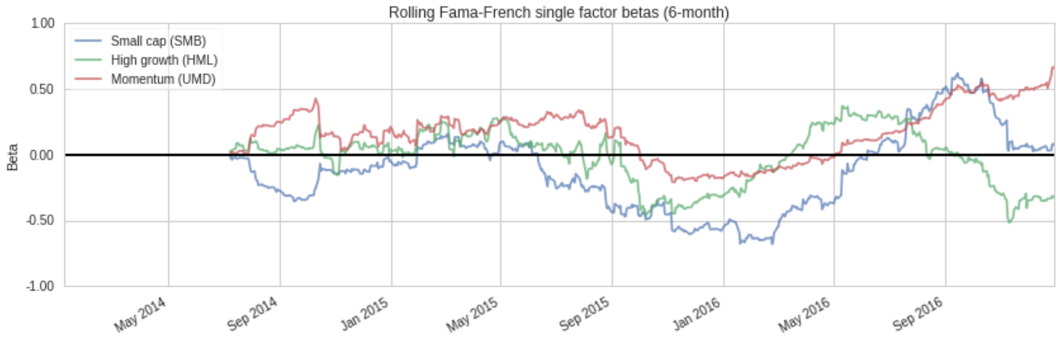

Cumulative returns on logarithmic scale
Equity(8554 [SPY])
Backtest
Cumulative returns
10.00
1.00
0.10
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

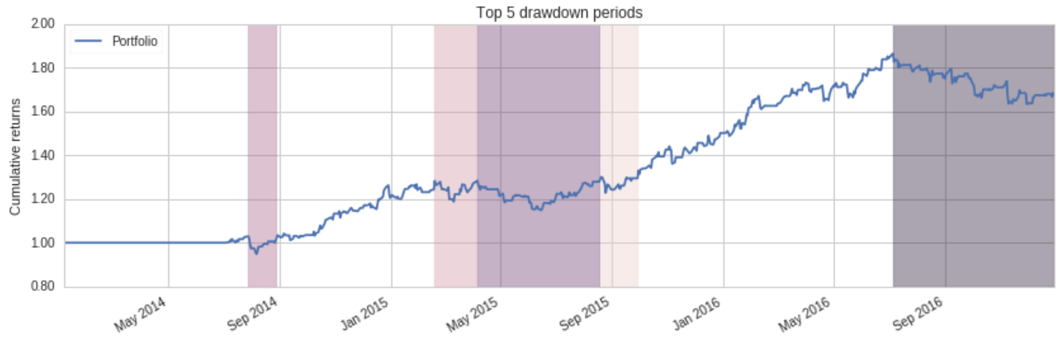

Returns
Returns
0.04
0.02
0.00
-0.02
-0.04
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

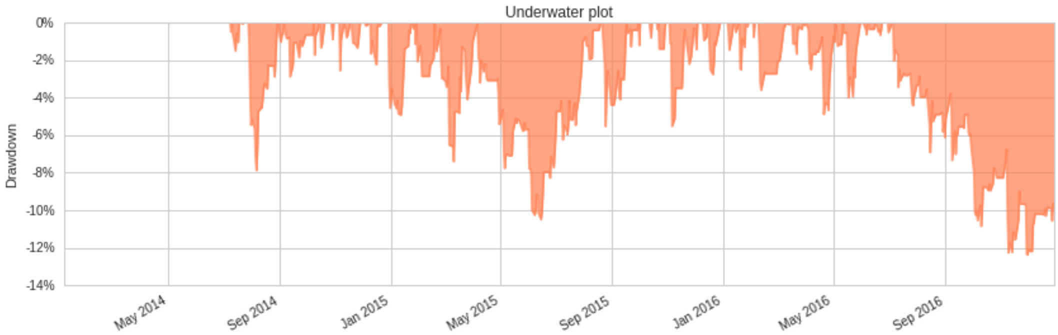

Rolling portfolio beta to Equity(8554 [SPY])
6-mo
12-mo
Beta
1.00
0.50
0.00
-0.50
-1.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

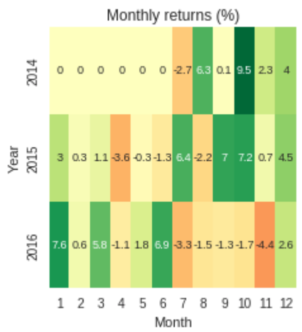

Rolling volatility (6-month)
Volatility
Benchmark volatility
Average volatility
Volatility
0.25
0.20
0.15
0.10
0.05
0.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

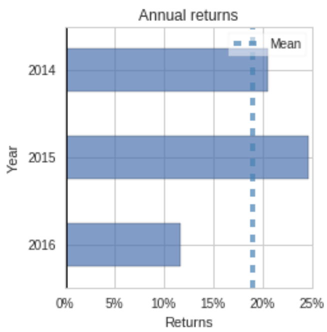

Rolling Sharpe ratio (6-month)
Sharpe
Average
Sharpe ratio
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

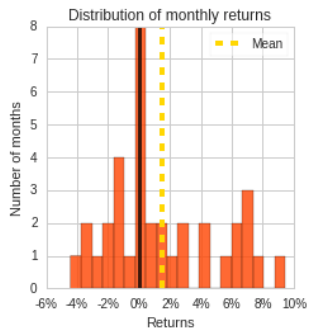

Rolling Fama-French single factor betas (6-month)
Small cap (SMB)
High growth (HML)
Momentum (UMD)
Beta
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

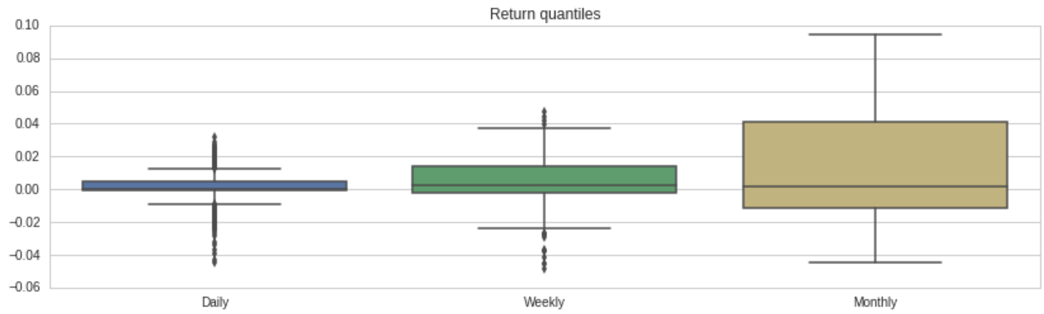

Top 5 drawdown periods
Portfolio
Cumulative returns
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016


Underwater plot
Drawdown
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

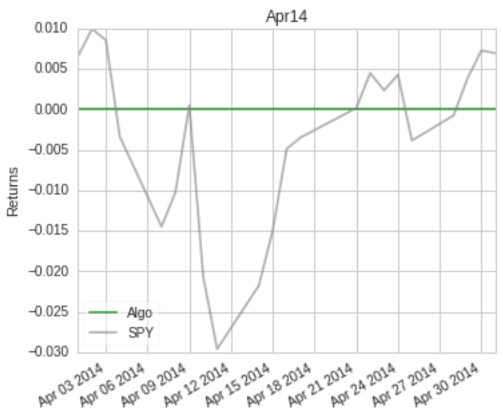

Monthly returns (%)
Year
Month

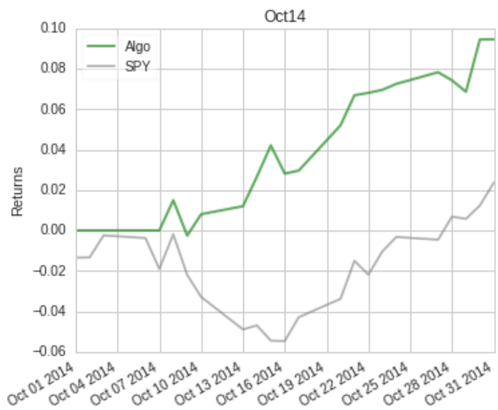

Annual returns
Mean
Year
Returns

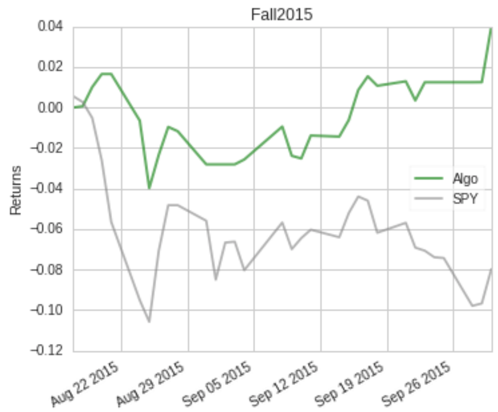

Distribution of monthly returns
Mean
Number of months
Returns

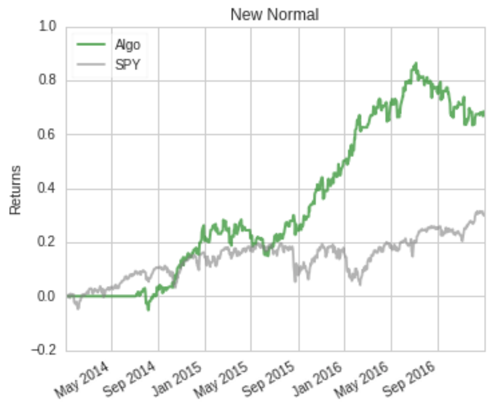

Return quantiles
Daily
Weekly
Monthly

Stress Events	mean	min	max
Apr14	0.00%	0.00%	0.00%
Oct14	0.40%	-1.73%	2.42%
Fall2015	0.13%	-3.35%	2.71%
New Normal	0.07%	-4.47%	3.16%







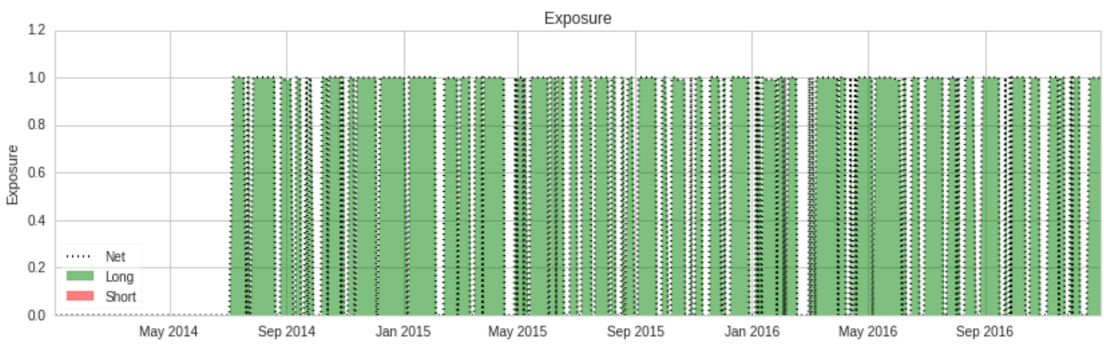


Top 10 long positions of all time	max
DTE-2330	100.26%
CMS-1665	100.21%

Top 10 short positions of all time	max

Top 10 positions of all time	max
DTE-2330	100.26%
CMS-1665	100.21%

All positions ever held	max
DTE-2330	100.26%
CMS-1665	100.21%

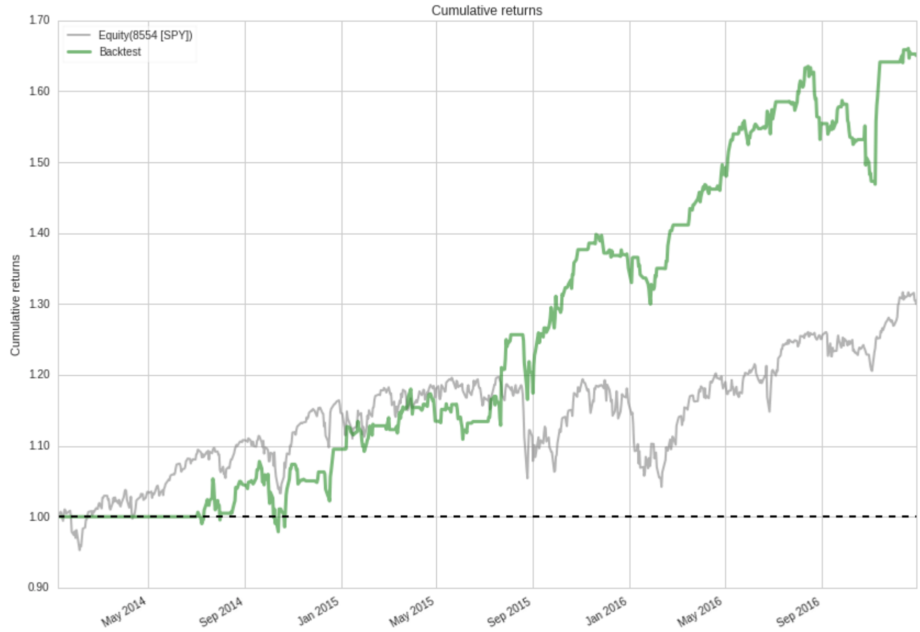


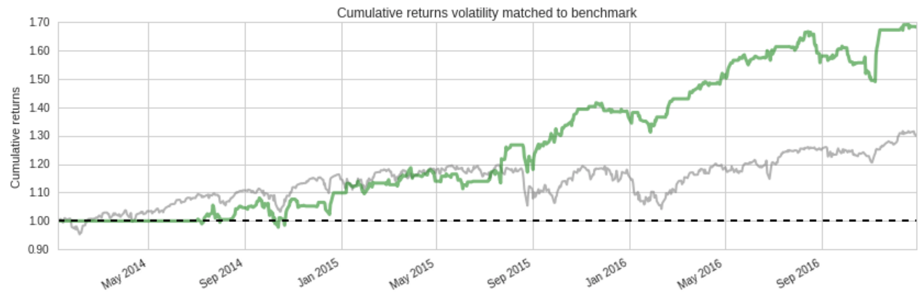
Full Tear Sheet of Pair RTN/LMT

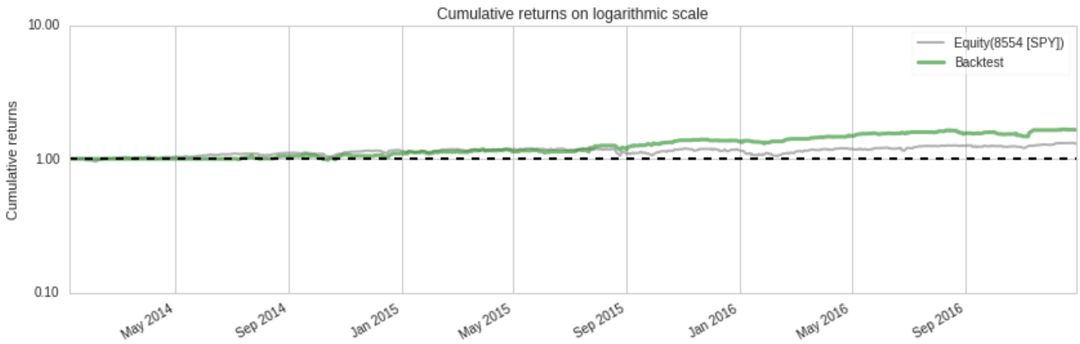
Start date	2014-01-06
End date	2016-12-30
Total months	35

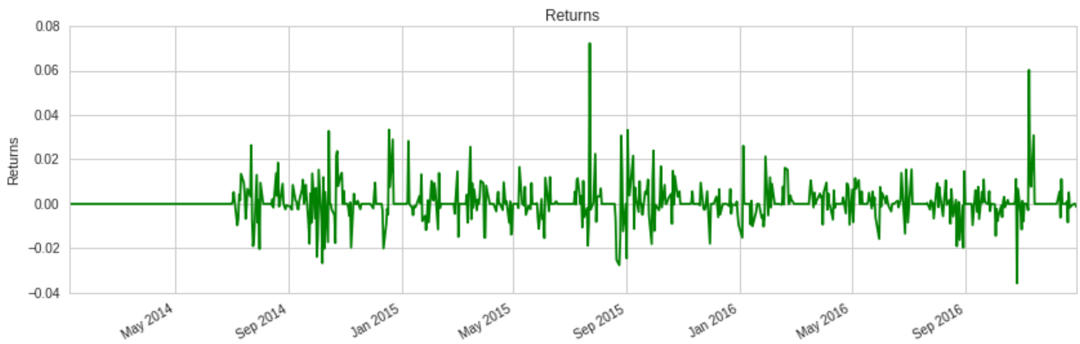
	Backtest
Annual return	18.2%
Cumulative returns	65.0%
Annual volatility	12.8%
Sharpe ratio	1.37
Calmar ratio	1.79
Stability	0.95
Max drawdown	-10.2%
Omega ratio	1.40
Sortino ratio	2.33
Skew	1.66
Kurtosis	14.44
Tail ratio	1.18
Daily value at risk	-1.5%
Gross leverage	0.45
Daily turnover	38.0%
Alpha	0.14
Beta	0.36

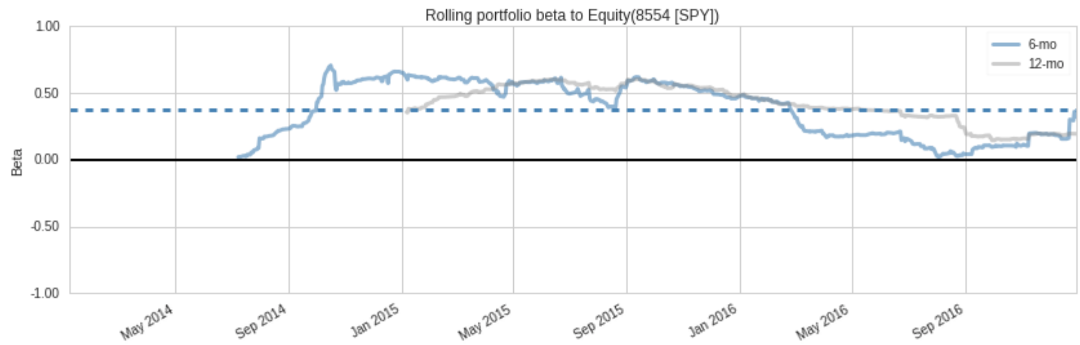
Worst drawdown periods	Net drawdown in %	Peak date	Valley date	Recovery date	Duration
0	10.17	2016-08-15	2016-11-08	2016-11-14	66
1	9.20	2014-09-19	2014-10-13	2014-12-22	67
2	7.27	2015-08-19	2015-08-25	2015-09-08	15
3	7.03	2015-11-20	2016-01-28	2016-02-22	67
4	6.01	2015-03-30	2015-06-04	2015-07-23	84

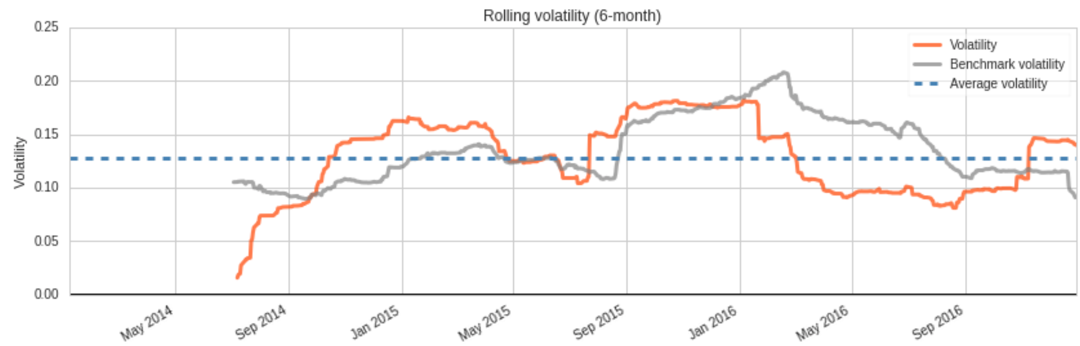


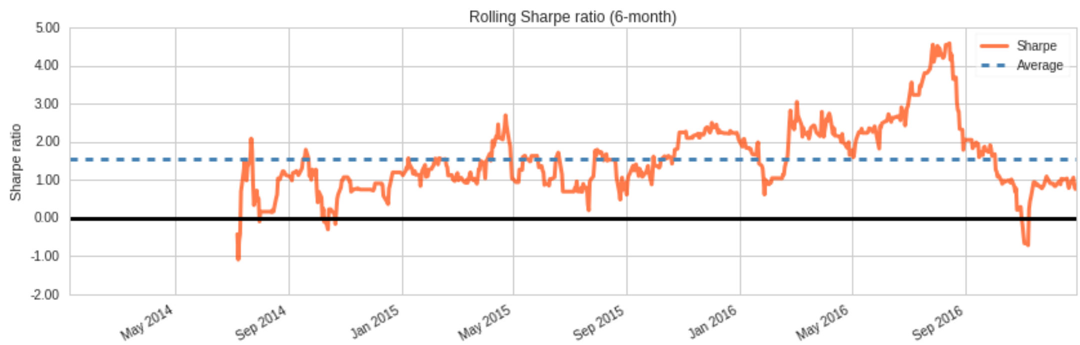


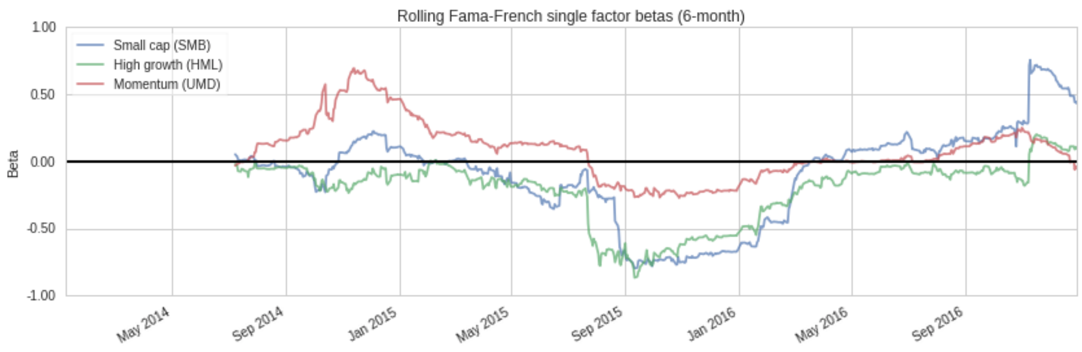

Cumulative returns on logarithmic scale
Equity(8554 [SPY])
Backtest
Cumulative returns
10.00
1.00
0.10
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

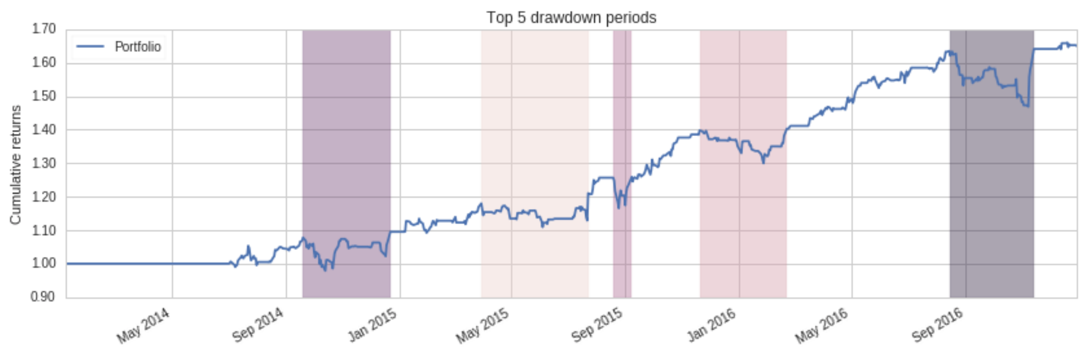

Returns
Returns
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

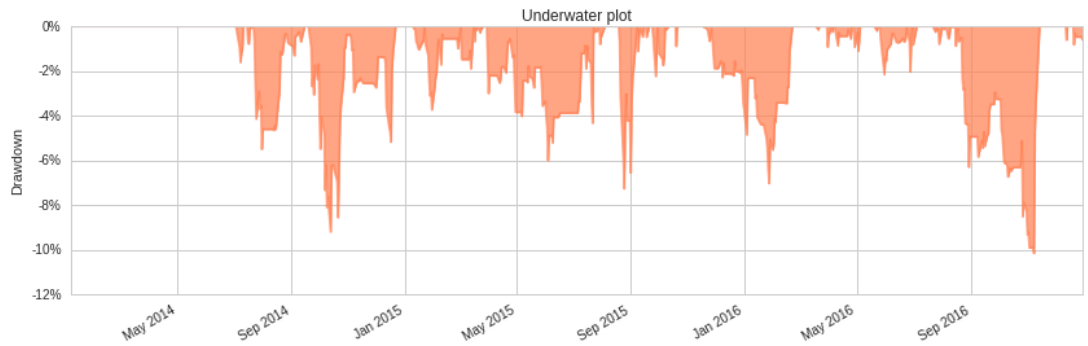

Rolling portfolio beta to Equity(8554 [SPY])
6-mo
12-mo
Beta
1.00
0.50
0.00
-0.50
-1.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

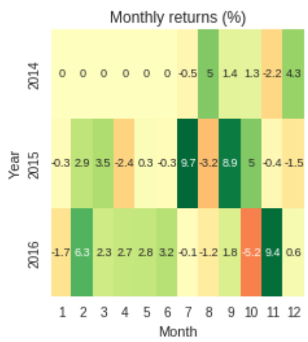

Rolling volatility (6-month)
Volatility
Benchmark volatility
Average volatility
Volatility
0.25
0.20
0.15
0.10
0.05
0.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

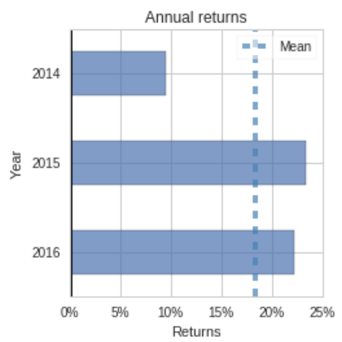

Rolling Sharpe ratio (6-month)
Sharpe
Average
Sharpe ratio
5.00
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

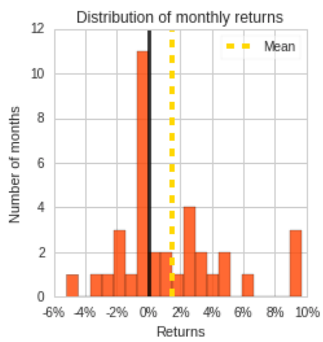

Rolling Fama-French single factor betas (6-month)
Small cap (SMB)
High growth (HML)
Momentum (UMD)
Beta
1.00
0.50
0.00
-0.50
-1.00
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

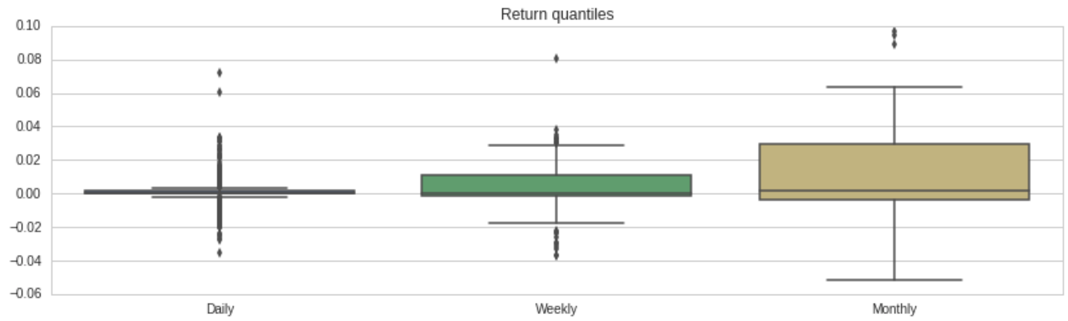

Top 5 drawdown periods
Portfolio
Cumulative returns
1.70
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016


Underwater plot
Drawdown
0%
-2%
-4%
-6%
-8%
-10%
-12%
May 2014
Sep 2014
Jan 2015
May 2015
Sep 2015
Jan 2016
May 2016
Sep 2016

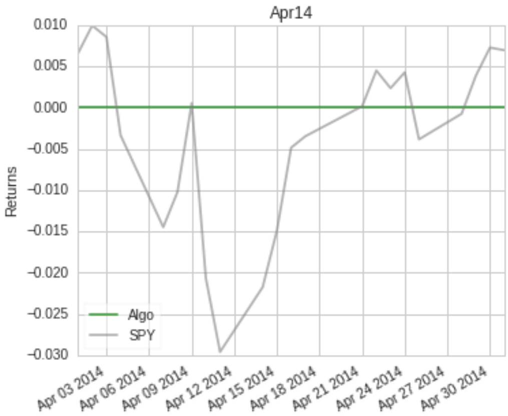

Monthly returns (%)
Year
2014
0 0 0 0 0 0 -0.5 5 1.4 1.3 -2.2 4.3
2015
-0.3 2.9 3.5 -2.4 0.3 -0.3 9.7 -3.2 8.9 5 -0.4 -1.5
2016
-1.7 6.3 2.3 2.7 2.8 3.2 -0.1 -1.2 1.8 -5.2 9.4 0.6
1 2 3 4 5 6 7 8 9 10 11 12
Month

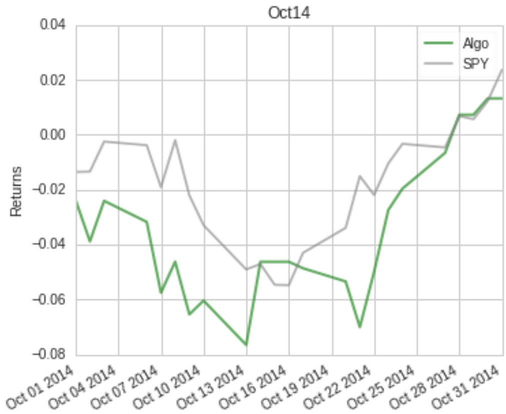

Annual returns
Mean
Year
2014
2015
2016
0%
5%
10%
15%
20%
25%
Returns

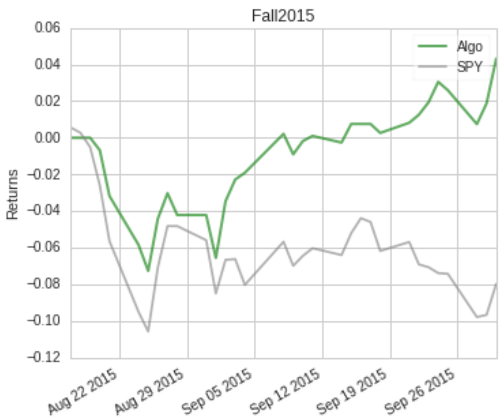

Distribution of monthly returns
Mean
Number of months
12
10
8
6
4
2
0
-6%
-4%
-2%
0%
2%
4%
6%
8%
10%
Returns

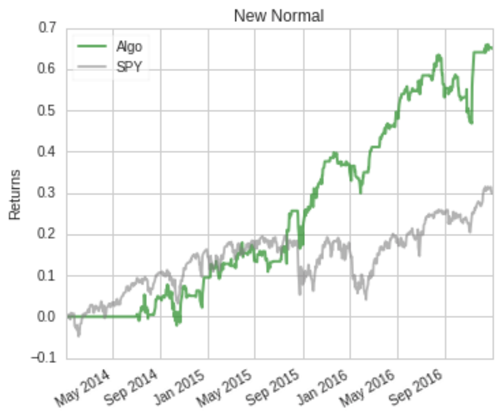

Return quantiles
0.10
0.08
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06
Daily
Weekly
Monthly

Stress Events	mean	min	max
Apr14	0.00%	0.00%	0.00%
Oct14	0.07%	-2.66%	3.28%
Fall2015	0.14%	-2.75%	3.32%
New Normal	0.07%	-3.57%	7.22%









Top 10 long positions of all time	max
RTN-6583	100.20%
LMT-12691	100.08%

Top 10 short positions of all time	max

Top 10 positions of all time	max
RTN-6583	100.20%
LMT-12691	100.08%

All positions ever held	max
RTN-6583	100.20%
LMT-12691	100.08%