5: Properties of regular languages

Properties of languages accepted by NFA's or DFA's.

Theorem 1 *The set of regular languages are closed under*

1. *Concatenation,*
2. *Union,*
3. *Kleene star,*
4. *Complementation.*
5. *Intersection, and*
6. *Reversal.*

Proof: 1, 2, 3, and 6 were proven earlier.

Item 4: Given that L is a regular language we will prove that $\overline{L} = \Sigma^* - L$ is also regular.

Let M be a DFA that accepts L. Let M' be the same as M except with different final states:

> A state is a final state in M' iff it is not a final state in M.

Then $\overline{L}$ is accepted by the DFA M' and hence is regular.

Item 5: Given that L_1 and L_2 are regular languages we will prove that $L_1 \cap L_2$ is also regular.
First note that

$$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}.$$

Applying Item 4 (three times) we find that $\overline{\overline{L_1} \cup \overline{L_2}}$ is regular, completing the proof.

Decision problems for finite automata

Theorem 2 *There exist algorithms for answering the following questions about finite automata:*

1. *Given FA M and string w, $w \in L(M)$?*
2. $L(M) = \emptyset$*?*
3. $L(M) = \Sigma^*$*?*
4. *Given FA's M_1 and M_2, $L(M_1) \subseteq L(M_2)$?*
5. $L(M_1) = L(M_2)$*?*

Proof:

1. Trace M on input w for $|w|$ steps.
2. Check if there is an path from s to f.
3. First construct M' such that $L(M') = \overline{L(M)}$ and see if $L(M') = \emptyset$.
4. Construct M_3 such that $L(M_3) = \overline{L(M_2)} \cap L(M_1)$ and see if $L(M_3) = \emptyset$.
5. Ask whether $L(M_1) \subseteq L(M_2)$ and $L(M_2) \subseteq L(M_1)$.