6: A Second Pumping Theorem

The pumping theorem we saw previously was:

Theorem 1 *Let L be an infinite regular language. Then there exist strings x, y, and z such that*

$$y \neq e, \text{ and } xy^n z \in L \text{ for all } n \geq 0.$$

We can also prove a stronger pumping theorem (Ex. 2.6.4):

Theorem 2 *Let $M = (K, \Sigma, \delta, s, F)$ be a DFA. Let $w \in L(M)$, such that $|w| \geq |K|$. Then there exist strings x, y, z such that*

1. $w = xyz$.
2. $|xy| \leq |K|$.
3. $y \neq e$.
4. $\forall n \geq 0, \quad xy^n z \in L(M)$

This second pumping theorem permits us to prove the non-regularity of many more languages than the first one can.

6: A Second Pumping Theorem

Example : $L = \{w = w^R : w \in \{a,b\}^*\}$ is not regular.

- Assume it was regular. Then there would exist a a DFA, $M = (K, \Sigma, \delta, s, F)$ with $L = L(M)$.
- Set $w = \overbrace{aaa\ldots aaa}^{|K|}\, b\, \overbrace{aaa\ldots aaa}^{|K|} = a^{|K|}ba^{|K|}$
- There exist strings x, y, z such that
  1. $w = xyz$.
  2. $|xy| \leq |K|$.
  3. $y \neq e$.
  4. $\forall n \geq 0, \quad xy^n z \in L(M)$
- 1. $x = a^p$.
  2. $y = a^q$, $q > 0$, $p + q \leq |K|$.
  3. $z = a^{|K|-p-q}ba^{|K|}$.
- Then, for every $n \geq 0$,

  $$a^{|K|+(n-1)q}ba^{|K|} = a^p(a^q)^n a^{|K|-p-q}ba^{|K|} = xy^n z \in L$$

- This leads to a contradiction. Thus L is not regular.